EXPLORE COMPUTING
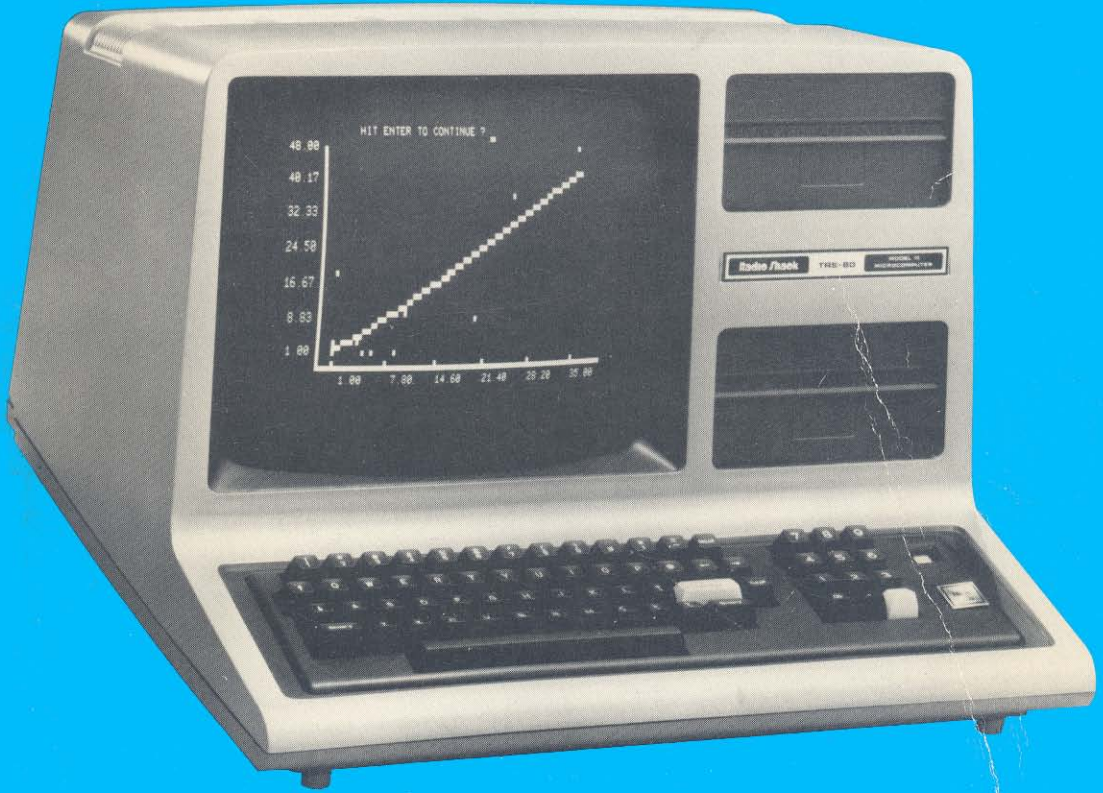## with the TRS-80
(& common sense)

with programming in BASIC

RICHARD V. ANDREE & JOSEPHINE P. ANDREE

$11.95

EXPLORE COMPUTING
with the TRS-80
(& common sense)

RICHARD V. ANDREE & JOSEPHINE P. ANDREE
University of Oklahoma

with programming in BASIC

Cover design by Mark A. Binn

Cover photograph courtesy of Radio Shack, a division of Tandy Corp.

CONTENTS

If you have never used a computer before, this introduction was written especially to help you gain familiarity with the keyboard. the screen and the use of BREAK , CLEAR , and ENTER keys. Your computer will do arithmetic and display messages. The main purpose of the introduction is to lift you from neophyte to beginner.

Beginners start here. A four-step program shows off your computer's speed as well as its versatility. Program instructions, PRINT variations, and additional keyboard commands are introduced.

Learn by programming the computer to create tables of squares, cubes, etc., for varying ranges and different step sizes. You are still not an expert, but you are well started.

You and your computer will solve problems that would be difficult, if not impossible, to solve without a computer. Nasty equations are easy to understand and to solve with the help of a simple computer program.

Although this lesson is mostly fun, the ideas developed help design
computer games and present graphical material in a dynamic format that
is easy to understant. Lots of room for fun and self-expression here.

Surprising as it may seem, you already have enough computing-know-how
to solve problems that would be too difficult to consider without com-
puter assistance. A bit of eighth-grade mathematics, common sense, and
your computer are all that is needed to investigate problems that would
intimidate a college student who did not have access to a computer or
calculator.

A review of the contents of the rest of the book—which may be taken up
in any order desired, or ignored, whichever serves your needs best.

As you develop longer programs you will wish to save the program on
magnetic tape using the cassette player that came with your computer.
Tape cassettes are a standard way of trading programs with other com-
puter owners. Cassettes are used for storing data and results as well
as programs.

Everyone loves to play computer games. Use your own ingenuity to create
new game programs and to improve programs written by others.

A serious programmer needs to know more about the powerful EDIT instruc-
tions available in your computer. This is where you will find answers.

College courses are given on SIMULATION, but the basic philosophy
involved is easy to understand and not difficult to program. This
introduction removes much of the mystery and enables you to program
your own simulations and computer games.

Additional BASIC instructions are available if and when you need them.
Turn here if you need to do something we haven't discussed.

Lesson 5 presented 17 micro-RESEARCH PROBLEMS you could explore with
the computing expertise available then. Here we discuss programming
practice, design and improvement of programs using easy-to-follow
examples. Lesson 17 then completes our collection of seventy-five (75)
choice micro-RESEARCH problems, each worthy of exploration, expansion,
and investigation. This collection contains a wide variety of proposals
including several arithmetical explorations, Haiku poetry, logical
decisions, calendar problems, dart and target games, loan and interest
programs, treasure hunt games, caricatures, a speech timer, puzzles,
recursive functions, polygonal maps, medical emergency prompter, num-
ber theory, lattice problems, magic prime squares, graphs, monkeys at
typewriters, amicable and sociable numbers, dance partners, and the
notorious eight queens problem. They are here for your entertainment
and education. Enjoy them.

PART I   **TO THE READER**

The first 100 pages carefully lead you from "watching the computer count fast" to where you have enough skill and confidence to write your own computer programs for creating random art, solving equations, or exploring microRESEARCH problems that interest you.

If you are a complete novice, start with the introduction.
If you are a beginner, start with Lesson 1. Work as many
    practice problems as you need.
If you are a bit more sophisticated, read the summary at
    the end of Lesson 1, work a few problems from Practice
    Session 1. Then continue with Lesson 2.
If you can already program in BASIC, read the summaries at
    the end of Lessons 1 & 2, work a few problems, and continue
    with Lesson 3.
Lesson 4 introduces graphics, computer assisted art and some
    of the special goodies available on the TRS-80.
Lesson 5 is devoted to solving problems that are easy to
    understand, but which would be very hard to solve with-
    out the assistance of a computer.

PART II

The first five Lessons will teach you most of what you need to know to use your computer in solving real problems.

The nature of Lessons 6 to 17 is different. These lessons are devoted to special topics you may or may not need to learn about, de- pending upon your own particular interests. Lessons 6 to 17 may be read in any order, or ignored entirely until needed. Consult the table of contents for topics covered.

The important thing is to HAVE FUN with your TRS-80. Remember, today's micro-computers are more versatile and more powerful than computers which cost 100 times as much, 20 years ago. Whether you purchased your TRS-80 as a playtoy or a serious tool, we hope you will soon be using it for both.

*   *   *   *

There are many different micro-computers, each with its own slightly different version of the BASIC computer language. Most of the instructions used in this book work on all versions of BASIC. The most frequent differences occur in the graphic display instructions (chapters 4 and 13) and in the string handling instructions (chapter 14). A chart is presented on pages 219 to 222 that displays the availability of various BASIC language instructions in the eight most common versions of the TRS-80, each of which is available with several different memory sizes and with different accessories.

TO THE INSTRUCTOR

This book is designed to be used by students for self-instruction (with or without a teacher's assistance). It has been successfully used by junior and senior high school students, secretaries, college students, 6th grade math classes, as well as a group of busy doctors and business people. Numerous examples are developed step by step to demonstrate the thinking involved. Since computing is a skill that must be learned by practice, a variety of problems are included. As with any acquired skill, a good coach can keep the neophyte interested and help develop good habits. Computing can and should be FUN. Students are fascinated by computers and will be responsive to your teaching.

Most readers are familiar with whole-number arithmetic, but may not have investigated the many tantalizing arithmetic puzzles that are easy to understand but hard to solve, unless a computer is available. A number of our problems come from the area of Number Theory. Problems like

1. Find squares like $(11)^2 = 121$ or $(264)^2 = 69696$ whose digits read the same forward as backward (palindromic squares).

2. Find squares such as $(35853)^2 = 1285437609$ in which all ten digits are present in the square.

are not difficult to program once the rudiments are learned (Lessons 1, 2,3). Each reader is encouraged to use common sense to write more effective programs. Lessons 4,5,7,9,13,14, and 17 provide numerous interest-catching problems from non-mathematical areas. We hope each reader will discover new interests and power as his programming skills develop.

Readers are encouraged to develop orderly (top-down, structured) programming habits without being preached at. Your assistance is sought in helping students develop this vital skill.

Lessons 5 and 17 present 75 micro-research problems that are easy to understand. Micro-research problems demand thoughtful program design to produce results with a reasonable investment of computer time. All have been successfully programmed by students with no more than 9th grade mathematical skill. In many cases, the programs "develop" as do the examples given in the text, starting first with a brute-force attack which could require weeks of computer time, and then gradually, as the early results are available for examination, refinements in program design (early trimming of logic trees) suggest themselves. Eventually, many students devise programs that produce more results in 5 minutes than the original program could produce in five days.

Appreciation and understanding of the value of skillful computing techniques are developed as needed, through experience.

The material has been used with Models I, II, & III TRS-80 micro-computers to teach introductory computing concepts to several different groups, including students in grades 5 through 14, adult neophytes having no experience on computers and little math beyond the 8th grade, secretaries and secretaries-in-training, a group of busy doctors, bankers and business people, and casual adult and child visitors at a museum. Each contributed to the presentation contained herein. Every vital concept is presented more than once, in different environments, to aid the student's comprehension. The important thing is to be sure readers also actively program computers, not passively read the text. Passive reading, like a spectator sport, does not develop skills. Encourage students to really program various kinds of problems.

With the exception of Lesson 4 ART & GRAPHICS and a few special goodies available only on the TRS-80 Level II BASIC, most of the material presented here is applicable to any computer using BASIC.

The first five lessons form a unit. Lessons 6 through 17 may be taken up in any order or ignored, as individual interests suggest.

Our experience suggests having two or three students per computer produces better results than one person per computer--particularly in the early stages.

Your authors welcome criticism, suggestions and correspondence.

<div align="right">

Richard V. Andree & Josephine P. Andree

The University of Oklahoma

Norman, Oklahoma   73019

</div>

ACKNOWLEDGMENTS

PART

I

The first five Lessons (100 pages) lead you from "watching the computer count fast" to enough skill to write your own programs to create computer art, solve messy equations or explore micro-research problems of your own choice.

If you are a complete novice
    start with the introduction

If you have used a computer keyboard but never written a program
    start with Lesson 1

If you are a bit more knowledgeable
    start with lesson 2

If you can already program in BASIC
    start with Lesson 3

If you understand BASIC, except for graphics
    start with Lesson 4

If you are a competent BASIC user
work a few problems from Lessons 3 and 4 and start with Lesson 5
which contains micro-research problems for your enjoyment.

Lessons 6 to 16 take up specialized topics discussed on page 103.

Lesson 17 continues with the micro-research problems begun in Lesson 5.

We assume you have no knowledge of computing, but are interested in learning. The way to learn computing is to compute!

Just as in golf or bowling, it is more helpful to read about computing *after* you have tried your hand at the game enough to have a feel for its rudimentary swing. So let's begin.

Let's start by using the computer as if it were a pocket calculator. It is a waste of the computer's power, but it will help you become familiar with the keyboard. Have someone turn it on for you. Depress the white [ENTER] key a couple of times.
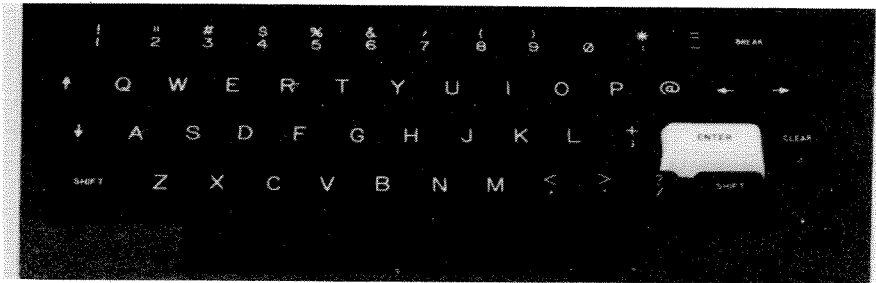
The keyboard is similar to a typewriter. If you wish to use a symbol like = or +   [=] [+]   from the top row of the key, depress the [SHIFT] key. The computer automatically types capital letters, as shown on the keys, so do *not* use the shift key unless you wish a symbol from the top half of a key.

Don't try to "read" this introduction—the way to learn computing is to compute, so wait until you have a TRS-80 at your fingertips before continuing.

3

Depress the following keys:

1. BREAK *(This interrupts the computer in case it was doing something.)*

2. ENTER *(This is the key you will use to send your typed message to the computer.)*

3. Now type: PRINT 3+5+9-2 ENTER

The computer should respond with the sum 15.

Your CRT-display will now show:

```
READY
>PRINT 3+5+9-2
15
READY
>_
```
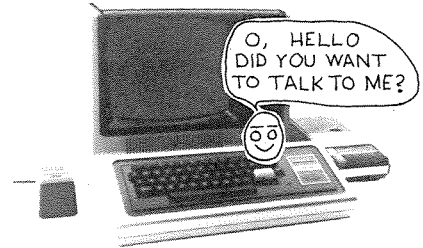
The > symbol indicates the computer is ready to receive your typed instruction.

The - shows where the next typed character will appear.

Should you wish to clear the screen, depress the CLEAR button, but there is no <u>need</u> to do so unless <u>you</u> wish it cleared.   You may also type CLS ENTER to <u>Cl</u>ear the <u>S</u>creen.

What will the answer be if you type

PRINT 4*9 - 2 ENTER

*Most computers use the symbols * for multiply and / for divide.*

Try it and see.

Forecast the result of each of the following, and then try them out on your TRS-80.

PRINT 4/3 ENTER
PRINT 9*5 ENTER
PRINT 360*18 ENTER
PRINT 1/2 + 0.6 ENTER

Note 0 is zero on the top row. O is the letter "oh" which is on the second row. Your computer will be unhappy if you confuse them. Try it once and see for yourself.

Let's try some hard problems
PRINT 774*817 ENTER
PRINT 79276/511017 ENTER
PRINT 317*1.41421 - 246*3.14159 ENTER

*If you type 774*817 ENTER without PRINT, the computer will do the arithmetic but will not print or store the result.*

4

The ⬅ key is another important key for those of us who occasionally mistype. It will backspace and erase one character at a time. If you hold down the SHIFT key while you depress ⬅ the entire line you typed will be erased. Try it and see for yourself.

Type

PRINT "HELLO " ENTER

Note the " " marks.
What happened?

Type

PRINT "SALUTATIONS " ENTER

PRINT "MY NAME IS *your name*" ENTER

Now let's try something different. This time we shall put a "line number" in front of each statement. First type   NEW   ENTER

Then type:

100 PRINT "HELLO " ENTER
110 PRINT "SALUTATIONS " ENTER
120 GOTO 100 ENTER

That isn't progress, or is it? Well, nothing happened. Did it? Yes, it did, but it happened inside the computer, where we can't see it, not on the screen.

Type

        LIST   ENTER

The computer should list the three statements you typed in. Check them to be sure they were properly entered with a space after each number and " " in the proper places.

Next type

    RUN   ENTER

After a bit, depress the BREAK key to get control again.
Depress the CLEAR button.
Type LIST ENTER
and the computer should respond by displaying:

    100 PRINT "HELLO "
    110 PRINT "SALUTATIONS "
    120 GOTO 100
    READY
    >_

It is perfectly reasonable to change these instructions by retyping them.

5

Type

    100 PRINT "HELLO *your name*"; [ENTER]   *Note the " " and the ;.*

    110 [ENTER]   *This erases the instruction in 110*
    as you will see when you type LIST [ENTER] *.*

Now type

    LIST   [ENTER]

The screen should show

    100 PRINT "HELLO *your name*";

    120 GOTO 100

Note that instruction 100 was changed and instruction 110 was deleted, but instruction 120 is unchanged.

Now type

    RUN   [ENTER]

    Let us write another program. Begin by depressing [BREAK] to get the computer's attention. Then type:

    NEW   [ENTER]
    100 X = 1980   [ENTER]
    110 Y = 365   [ENTER]
    120 Z = X*Y   [ENTER]
    130 PRINT X; Y, Z   [ENTER]
    140 GOTO 100   [ENTER]
    RUN   [ENTER]

The above program sets X=1980, then sets Y=365, then forms the product of the values of X and Y and stores it in a location called Z. Instruction 130 then prints out the values of the numbers stored in locations X, Y, and Z. Instruction 140 then sends the computer back to instruction 100, which repeats the entire process.

    Depress the [BREAK] key to stop the computer.

Let's see what you have learned thus far...

To get the computer's attention, depress BREAK key.

Once you get the computer's attention, which it indicates by displaying

```
READY
>_
```
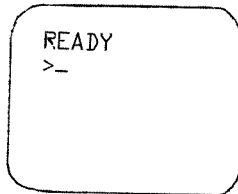
on the CRT (TV screen), you can work arithmetic problems involving +, -, *, / (add, subtract, multiply, divide) by typing PRINT followed by the arithmetic problem.

When the problem is displayed the way you wish it, depress ENTER to send it to the computer. Remember, nothing goes to the computer until you depress ENTER .

Oh, yes. If you make an error in typing, just depress the ⟵ key to backspace and erase. You can do it anytime before you depress the ENTER key.

To clear the screen, depress the CLEAR key, followed by ENTER if the >_ symbol is absent, or type CLS ENTER .

You need to be careful about confusing zero, $\emptyset$, and the letter oh, O. Also, 1 and L are distinct.

You have written a set of instructions to the computer, and the computer has followed these instructions.

Now that you no longer fear the TRS-80, let's begin our lessons.

LESSON 1
Make Your TRS-80 Work
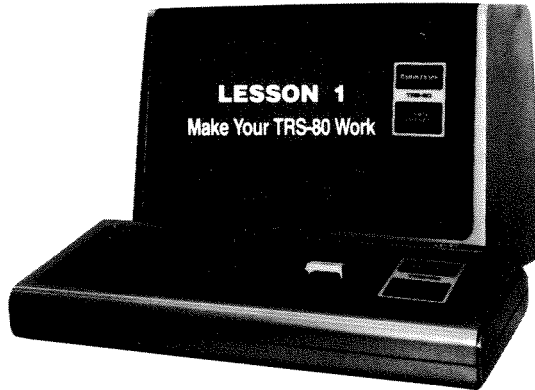
So you don't know anything about computers or computing. Well, this won't make you an expert, but let's see what we can do.

Someone has connected and assembled the TRS-80 and turned it on for you. The screen says something on it. I don't know what because I don't know what was done last.

If your TRS-80 has just been turned on, the screen may display

MEMORY SIZE?
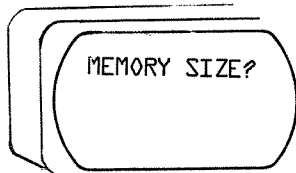
The TRS-80 can set aside memory for programs written in languages other than BASIC, but we shall not do so.

If so, just depress the white ENTER key. The screen will then display an additional message, which should end with
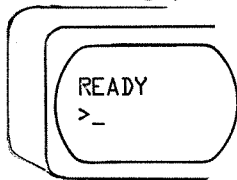
READY
>_

The word READY is your clue that the computer is ready to accept instructions from you.

To clear up everything...

Depress the BREAK key (upper right).
The computer will add

```
READY
>_
```
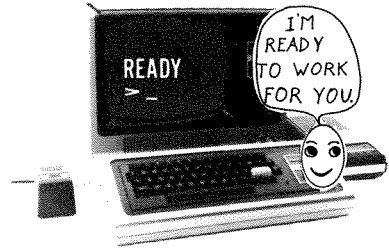
below whatever was on the screen. Now you have its attention. The >- symbol below READY means the TRS-80 is waiting for you to type in an instruction.

Type NEW and depress the white ENTER key.

Everything on the screen disappears—also any program inside disappears too—the slate is wiped clean for you, and the computer displays

```
READY
>_
```
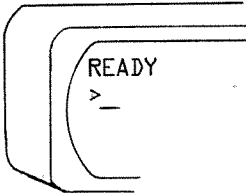
NOTE: *Nothing goes from the screen to the computer until you depress the* ENTER *key.*

Now type

    21Ø X = 1   ENTER

The screen should now show
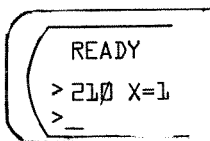
If you mistype, just use the ← *button to backspace and erase--or depress* BREAK *and start the entire line over.*

```
READY
> 21Ø X=1
>_
```

The symbol >_ indicates the TRS-80 is ready to accept another instruction, so type

    22Ø PRINT X;   ENTER   *Don't forget the ;*

The screen now shows

```
READY
> 21Ø X=1
> 22Ø PRINT X;
>_
```

9
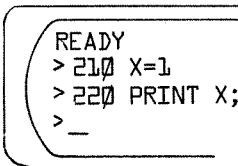
Continuing, type

>   23Ø X = X + 1 |ENTER|
>   24Ø GOTO 22Ø |ENTER| *Note that GOTO is one word.*

To make sure the program you typed is really there, type

>   LIST |ENTER|

Your program will then be listed below whatever is already on the screen.

```
READY
>21Ø X=1
>22Ø PRINT X;
>23Ø X=X+1
>24Ø GOTO 22Ø
>LIST
21Ø X=1
22Ø PRINT X;
23Ø X=X+1
24Ø GOTO 22Ø
>_
```

I'LL BE GLAD
TO SHOW
YOU
THE PROGRAM.

Note the absence of the > symbols on the LISTed program, indicating the computer typed it to you.

Now you are ready to run your first numerical program. Type RUN |ENTER| and watch what happens.

You probably have a Level II BASIC, but if not, read the right-hand column instead of the left-hand column below.

Level II BASIC

If you hold down |SHIFT| and depress |@|, the program will halt until you depress another key (without |SHIFT|). Try it a few times and see.

Level I BASIC

If you depress the |↑| key on left edge of the keyboard, the program will halt whatever it is doing, as long as you keep the |↑| key depressed, but will continue whenever you let it up. Try it a few times and see.

In either case, if you wish an extended halt, depress the |BREAK| key. (Do so now.) Not only does the TRS-80 stop, but it even tells you which instruction it was executing when you depressed |BREAK|. If you want the program to continue, simply type: CONT |ENTER| and it will continue.

Note that in Level I BASIC, if the program is typing a number, say 425 or 438, and runs out of line, it prints part of the number at the end of one line and the rest of the number at the beginning of the next line. The output is difficult to read, but the data is printed quickly and the screen displays a lot of data at once.

10

Level II BASIC is a bit more sophisticated and will leave the end of the line blank, rather than break a number, but it still changes the column spacing as the number of digits in the number changes in order to get as much output on a line as is convenient. The ; at the end of
    22Ø PRINT X;
is what controls the "packed spacing" of this output.

Depress [BREAK]

Type RUN [ENTER]

to see what is meant.

Let's look at our program again.

Depress the [BREAK] key and the [CLEAR] key.

Then type LIST [ENTER]

The computer should respond.

```
>LIST
 21Ø X=1
 22Ø PRINT X ;
 23Ø X=X+1
 24Ø GOTO 22Ø
 READY
>_
```


NOW I MUST ADD 1 TO WHATEVER NUMBER IS IN STORAGE LOCATION X, AND STORE THE NEW BIGGER NUMBER BACK IN X LOCATION
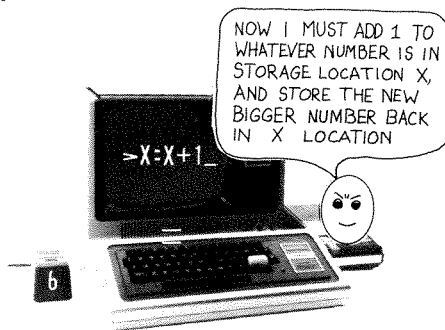
Instruction 23Ø X=X+1 looks odd. If it were a mathematical equation, it would have no solution. It is not a mathematical equation; it is an instruction to a computer.

X=X+1   This instructs the computer to "Take the number in the storage location called X, add 1 to it, store the new result back into the storage location called X."

X=X+1   could be more reasonably written as X←X+1. However, it is almost universal computer practice to use = rather than ← , probably because old fashioned typewriter keyboards contain = not ← .

Let's change instruction 22Ø PRINT X; by changing the semicolon to a comma. To do this, we simply type

    22Ø PRINT X, [ENTER]

If you now type

    LIST [ENTER]

the new program will be displayed below the LIST instruction. Please do so before continuing. Then type RUN [ENTER] .

11

Note that now our program prints out its values in four nice columns.

Isn't it impressive?

You may wonder what would happen if you did not have either the comma or the semicolon after PRINT X. If so, experiment a bit and find out.

Depress ⌈BREAK⌉ to get the computer's attention.

Type LIST ⌈ENTER⌉

Type 22Ø PRINT X ⌈ENTER⌉

Type LIST ⌈ENTER⌉

Type RUN ⌈ENTER⌉

Depress ⌈ BREAK ⌉ to get the computer's attention.

Do some other experiments on your own. For example, change 23Ø X=X+1 to
23Ø X=X+.5 or 23Ø X=X+2 and see what happens when you type

  RUN ⌈ENTER⌉.

Remember, about the only thing you
might do that would hurt the TRS-80
would be to drop part of it or spill
a beverage over it. Otherwise, it is
pretty rugged, as long as you don't
open the case.

KEEP YOUR COKES
AWAY! SPILLS ARE
HAZARDOUS TO
MY HEALTH.
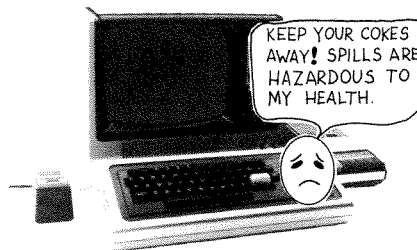
A Word of Warning:
""""""""""""""""""""""""

     It is always well to check that you have typed what you think you
have typed before depressing the ⌈ENTER⌉ key... In most cases it is
easy to correct an error by simply retyping that statement number and
instruction, but if you should happen to have typed LLIST ⌈ENTER⌉
instead of LIST ⌈ENTER⌉ you may have a problem you will need to call
your instructor to fix. LLIST is also a perfectly valid instruction--
it tells the TRS-80 to list your program on the attached line printer--
but if your TRS-80 has no line printer attached (and turned on), the
computer will not accept another instruction until it executes the im-
possible instruction LLIST ⌈ENTER⌉ . To all appearances your computer
will "lock-up." Your instructor can get you out of this difficulty
without having to turn off the computer. If you are without an instruc-
tor, open the small door on the back left top side of the keyboard, and
press the button inside.

If your TRS-80 has "keyboard bounce" (printing multiple letters on one keystroke), it means the contacts are dirty and no one inserted the "keyboard fix" program when the TRS-80 was turned on. Ask for help if you have excessive troubles with keyboard bounce. Each TRS-80 has a keyboard-fix tape and manual packed with it. If you do not have an instructor, consult the manual that accompanies the keyboard fix tape.

H A V E   F U N ! ! !

SUMMARY OF LESSON 1

Let's see what you have learned thus far...

Depress ⎡BREAK⎤ key to get computer's attention if it is busy.

Type NEW ⎡ENTER⎤ to clear out everything and start over.

Number each instruction and put a space between the number and its instruction.

(If you fail to number an instruction the TRS-80 will perform that instruction right then, but not store it for later use. Try typing PRINT X, Y, T, L ⎡ENTER⎤.*)*

You can erase errors by using the ⎡◄—⎤ key as a "backspace" if you have not depressed ⎡ENTER⎤.

If you have depressed ⎡ENTER⎤ it's easy to change instructions, merely retype the number and new instruction. You may insert additional instructions between those already in use by using statement numbers that fall between those already used.

Holding down ⎡SHIFT⎤ while you depress ⎡◄—⎤ erases the entire line.

The symbol >_ indicates the computer is ready to receive typed in-
structions from you.

If the >_ symbol is not there, depress ENTER or BREAK   ENTER .

In the PRINT instruction, the use of a semicolon ; between or after
variables will give a "packed format", while the use of a comma,
produces four columns. No symbol gives one output per line (wasteful).

You may halt the output (temporarily) by depressing the ⬆ if you
have a Level I.   If you have a Level II, hold down SHIFT and depress
 @ . Depress any key except SHIFT   or BREAK   to continue.

LIST   ENTER produces a listing of the program currently in storage.

Among your output you may discover some strange-looking numbers, such as
1.23456E+$\emptyset$8 or 2.22222E-$\emptyset$5. The computer displays only six significant
decimal digits, and shows the magnitude by appending an E+nn or E-nn to
indicate the power of ten by which the preceding fraction should be
multiplied.

Thus

$1.23456E+\emptyset 8 = 1.23456 * 1\emptyset^{+\emptyset 8} = 123456\emptyset\emptyset\emptyset.$ *(But recall this may be in
error by ±5 in the seventh
place, that is, by ±5$\emptyset\emptyset$.)*

$2.22222E-\emptyset 5 = 2.22222 * 1\emptyset^{-5} = .\emptyset\emptyset\emptyset\emptyset 222222$ *(±.$\emptyset\emptyset\emptyset\emptyset\emptyset\emptyset\emptyset\emptyset\emptyset 5$)*
$3.45678E+11 = 345678\emptyset\emptyset\emptyset\emptyset\emptyset$
$-4.68\emptyset 24E-\emptyset 7 = -.\emptyset\emptyset\emptyset\emptyset\emptyset\emptyset 468\emptyset 24$

This convenient notation, a cousin of the so-called "scientific notation,"
is used on most modern computing systems, including many hand-held cal-
culators.

PRINT X,Y;Z;T will print <u>values</u> of X   Y Z T , spaced as shown.

 CLEAR or C L S   ENTER will clear the screen.

 SHIFT   ➡ will produce double size letters on the screen (see
problem 13).

Usually CLEAR is depressed <u>just before</u> using SHIFT   ➡ .

Problems 1 to 6 make use of the program discussed in the text. Start by typing it in, LISTing the program and RUNning it to check that it is functioning properly before you start the problem set.

depress BREAK key, then type:

```
        NEW   ENTER
        21Ø   X = 1   ENTER
        22Ø   PRINT X;   ENTER
        23Ø   X = X + 1   ENTER
        24Ø   GOTO  22Ø   ENTER
```

LIST   ENTER

RUN   ENTER

BREAK

CONT   ENTER

BREAK

Now you are ready to start the practice set.

1. Type in the program (using 22Ø PRINT X;) first given in this lesson. Run it a few times with 23Ø X=X+1 replaced by each of the following in turn.

```
        23Ø   X=X+.5   ENTER
    or  23Ø   X=X+ 5   ENTER
    or  23Ø   X=X+ 2   ENTER
    or  23Ø   X=X+.1   ENTER

    Type  LIST   ENTER
          RUN    ENTER
```

each time you change instruction 23Ø. Note any peculiar behavior and think about it a bit. Discuss it with your instructor or a fellow computer buff.

15

2. In the program above, replace 21Ø X=1 with 21Ø X=7 and rerun the program.

3. Forecast the output of the program.
   ```
   21Ø   X=1
   22Ø   PRINT X,
   23Ø   X=X+X
   24Ø   GOTO  22Ø
   ```

   Try it out and test your forecast. What about column spacing?
   The error message ?OV ERROR IN 23Ø indicates that the arithmetic operation in instruction 23Ø produced a result too large for the computer to handle--namely larger than 10^{38} --which is a number larger than the total number of atoms in the solar system.

4. What output would you expect the program of problem 3 to produce if 24Ø GOTO 22Ø were replaced with 24Ø GOTO 21Ø?

   Try it out and verify your forecast, or explain why the computer did not do as you forecast.

   What would happen if you used 24Ø GOTO 23Ø?
   Be sure you understand these points.

5. Let's try another program. Depress BREAK if needed. Then type NEW
   If you know that most computing systems use * to indicate multiplication, you should be able to forecast what the following program will produce.
   ```
   21Ø   X=2
   22Ø   PRINT X,
   23Ø   X=X*X
   24Ø   GOTO  22Ø
   ```

   Make a forecast. Then run the program. If the results are unexpected, reread problem 3.

6. Replace instruction 22Ø PRINT X, in problem 5 with 22Ø PRINT X, X+5 and see what happens. Before running the program, try to forecast the output of the first 5 lines.

7. Try some experiments on your own. Don't be afraid--you are not apt to harm the TRS-80 computer providing you don't eat or drink in the computer room and don't move or bump it while it is connected. Be sure to depress BREAK and type NEW   ENTER before you start a new program.
   The new instructions used in problems 8,9,10 will be discussed in detail later. Now all you need to know is that
   IF RND(Ø)<.Ø5 THEN CLS is pronounced "If 'random number'
   is less than .Ø5 then clear the screen."

16

8. Let's try another program. Depress [BREAK] if needed. Then type
   NEW [ENTER].

   Level II BASIC

   1∅ PRINT @ RND(1∅∅∅), "HI *type your name*"; [ENTER]
   15 IF RND(∅)<.∅5 THEN CLS [ENTER]
   2∅ GOTO 1∅ [ENTER]

   RUN [ENTER]

   Level I BASIC

   *If you have a
   Level I BASIC re-
   place* [@] *with*
   [A] [T] *and it
   will behave
   similarly.*

9. Experiment a bit by changing instruction 15 IF RND(∅) < .∅5 THEN
   CLS to <u>one</u> of those suggested below.

        15 IF RND(∅) <.∅1 THEN CLS
   or   15 IF RND(∅) <.1 THEN CLS
   or   15 IF RND(∅) <.5 THEN CLS
   or   15 CLS

   [Don't forget to depress [ENTER]
   *each time you change an in-
   struction.]*

10. Try combining the program in the text with that of problem 8, with
    instruction 2∅ omitted, giving:
        1∅ PRINT @ RND(1∅∅∅), "HI *type your name*";
        15 IF RND(∅) < .∅5 THEN CLS *(Type 2∅* [ENTER] *to delete instr. 2∅.)*
        21∅ X=1
        22∅ PRINT X;
        23∅ X=X+1
        24∅ GOTO 22∅

11. Change the instruction   24∅ GOTO 22∅ to 24∅ GOTO 1∅ in problem
    10. Forecast the output before you RUN the program. If your fore-
    cast was not valid, try to find a why before reading problem 12.

12. Delete instruction 21∅ X=1 by typing 21∅ [ENTER]. When >_

    appears, insert 5 X=1. Type LIST [ENTER] to check your new pro-
    gram. Before you RUN the program, try to forecast the result. Why
    is it different than that of problem 11 which used the same in-
    structions, but placed differently in the program?

13. If you have a Level II BASIC, please try the following sometime
    when you have a program that does not contain CLS in the computer.

    Depress [BREAK]   if needed.
    Depress [CLEAR]
    Hold down [SHIFT] and depress [⟵]
    Type LIST   [ENTER]
    Notice that your program is listed in characters <u>twice</u> as large as
    before.
    Now type   RUN [ENTER]
    The output is also double size. This is handy if you are using the
    computer with a class or group. Double size will last until [CLEAR]

key is depressed or until you execute a <u>CLS</u> instruction. To set the
regular size back, depress | BREAK | and then | CLEAR |.
If you fail to clear the screen before you depress | SHIFT | | ⟹ |,
your double-size letters and numbers obliterate half of the material
on the screen to make room for the larger letters, but any material
printed after that will be complete.

14. Have fun. Make your own variations.
     Forecast the result before you run each program.
     Try NEW | ENTER |

         21∅ X = 5 | ENTER |

         22∅ PRINT X; X+5; X+1∅; X+15; | ENTER |

         23∅ X = X+2∅ | ENTER |

         24∅ GOTO 22∅ | ENTER |

     Now investigate what happens if you omit the final ; in line 22∅.
     What happens if you change the semicolons to commas in line 22∅
     (with and without the final comma)?

15. Can you write a program that will start with X = 1 and count by
     <u>threes</u>, with exactly four numbers on each line? There are several
     correct ways to write such a program, so when you are finished,
     compare your program with that of a friend who also has a computer.

16. Try the following program.

         NEW | ENTER |

         1∅∅ X = 2 | ENTER |

         11∅ PRINT X, X*X, X*X*X, X*X*X*X | ENTER |

         12∅ X = X+1 | ENTER |

         13∅ GOTO 11∅ | ENTER |

         RUN | ENTER |

17. The following floating-point numbers are expressed in regular decimal
     notation with the possible error indicated:

FLOATING-POINT NUMBER		DECIMAL VALUE	POSSIBLE ERROR
1.23456E+∅6	=	1234560	± 1∅
2.13145E-∅3	=	.∅∅213145	±.∅∅∅∅∅∅∅∅5

     Do the same for:

         9.87654E+∅7   -2.34567E-∅7
        -1.23456E+∅8   5.67898E+15
         4.56789E-∅4   4.32198E+11

18
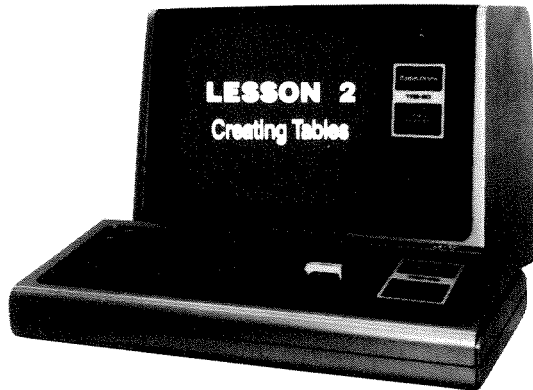
In our first lesson, we learned to make the computer
count, using various step sizes
to print in different formats, depending upon whether we used
     PRINT X
 or   PRINT X;
 or   PRINT X,

We also learned the essential handling of the computer using BREAK ,
ENTER and ◄━━━ "backspace" keys
and to type NEW ENTER   to clear out old program and data, ready to
accept new ones.

We rewrite the counting program using a slightly different philosophy
below (for reasons you will soon appreciate.) We shall use B = begin-
ning value, S = stepsize by which X will be increased, and F = final
value, after which we wish the computer to stop.

```
NEW   ENTER
 2    B=1  ENTER
 4    S=1  ENTER
 6    F=1ØØ  ENTER
1ØØ   X = B  ENTER
11Ø   PRINT X;  ENTER
12Ø   X = X+S  ENTER
13Ø   IF X < F THEN 11Ø  ENTER
```

Instruction 13Ø examines the current value of X and if X is less than
F, sends the program back to instruction 11Ø. If you feel at all in-
secure about this program, put it on the computer and RUN it before
continuing.

19

The real advantage of this technique is that we can easily change the step size by changing instruction 4 to 4 S = .5 [ENTER]

Indeed, we may rewrite the program slightly to input values of B, S, and F from the keyboard instead of assigning them in instructions 2, 4 and 6.

We do so next, using the instruction INPUT B,S,F . When the computer executes this instruction it will display a question mark on the TV screen and wait for you to type in three values, separated by commas. The values will be assigned, in the order typed, to variables B,S, and F. These letters were used as they are the initial letters of the phrases Beginning value, Step size, and Final value.

The revised program is:

```
NEW  [ENTER]
 9Ø   INPUT B,S,F  [ENTER]
1ØØ   X=B  [ENTER]
11Ø   PRINT X;  [ENTER]
12Ø   X=X+S  [ENTER]
13Ø   IF X < F  THEN 11Ø  [ENTER]
14Ø   PRINT  [ENTER]
15Ø   PRINT  "END OF TABLE WITH B,S,F=";B;S;F  [ENTER]
16Ø   PRINT :  PRINT  [ENTER]
17Ø   GOTO 9Ø  [ENTER]
RUN  [ENTER]
```

Again, note the use of instruction

13Ø IF X < F THEN 11Ø.

As long as X is less than F, the program loops back to 11Ø PRINT X; but when X ≥ F, the program continues with statements 14Ø, 15Ø, 16Ø, and 17Ø which upspaces the printed matter, prints an END OF TABLE message, and then sends the computer back to 9Ø for a new set of input values.

Instruction 15Ø PRINT "END OF TABLE WITH B,S,F =";B;S;F is particularly interesting. It combines the printing of a message in quotes, " " with the printing of the values of the variables B,S and F, which must not be included within the quotes.

Try this program on your TRS-80 before continuing. Begin by typing RUN [ENTER] . When a ? appears on the screen, type 1, 1, 1ØØ [ENTER] . Watch the output. When a ? appears at the bottom of the screen, type 2, .5, 1Ø [ENTER] or some B, S, F values of your own choice.

Let's write a program to create a table of values of

X X^2 X^3 X^4

When our program is run, it will accept values B, S, F from the keyboard. It will set X=B and print out values of X, X^2, X^3, X^4 for that value of X. Then the instruction X=X+S will add S to the current value of X and print another set of X, X^2, X^3, X^4 for the new value before adding another S to the current X.

This loop will continue until X exceeds the value of F (X>F) in which case the program will PRINT a blank line (16∅) followed by (17∅) an END OF TABLE message.

```
    One way to do it is:
NEW    ENTER

1∅∅  PRINT "PLEASE TYPE BEGINNING VALUE, STEP SIZE, FINAL VALUE "; ENTER
1∅5  INPUT B,S,F  ENTER
11∅  X=B  ENTER
12∅  Y=X*X  ENTER
13∅  PRINT X, Y, X*Y, Y*Y  ENTER
14∅  X=X+S  ENTER
15∅  IF X < = F THEN  12∅   ENTER
16∅  PRINT  ENTER
17∅  PRINT "END OF TABLE"  ENTER

RUN  ENTER
```

Let us examine the program in detail.

The instruction

1∅5 INPUT B,S,F

causes the computer to type a ? and wait for the user to type three values separated by commas and depress the ENTER key.

Instructions 1∅∅ and 1∅5 can be combined into a single statement, but we shall not do so yet.

Instructions

11∅ X=B
12∅ Y=X*X
13∅ PRINT X, Y, X*Y, Y*Y

do just what you learned to expect in lesson one. Note that by using commas between output, we obtain four neat columns. Also note that the use of Y=X*X saves half of the multiplications that would be required by
PRINT X, X*X, X*X*X, X*X*X*X   since Y*X = X^3 and Y*Y = X^4.

The instruction

14∅ X=X+S

merely increases the current value of X by the amount in S.

21

Next we use the branch instruction

15Ø   IF X<=F   THEN 12Ø
      This instruction compares the values stored in X and in F. If the
      value in X is less than or equal to (<=) the value in F, the pro-
      gram loops back to instruction 12Ø. Otherwise the instruction which
      follows instruction 15Ø (namely 16Ø PRINT) is executed next.

16Ø   PRINT
17Ø   PRINT "END OF TABLE"

      Instruction 16Ø merely prints a blank line, while instruction 17Ø
      prints the message indicated in quotes. If after entering the pro-
      gram, you depress CLEAR type LIST ENTER , the computer display
      will show

```
>   LIST
1ØØ   PRINT "TYPE BEGINNING VALUE, STEP SIZE, FINAL VALUE";
1Ø5   INPUT B,S,F
11Ø   X=B
12Ø   Y=X*X
13Ø   PRINT X, Y, X*Y, Y*Y
14Ø   X=X+S
15Ø   IF X<= F   THEN 12Ø
16Ø   PRINT
17Ø   PRINT "END OF TABLE"

READY

>__
```

If you now type RUN ENTER , the computer will respond

> RUN
TYPE BEGINNING VALUE, STEP SIZE, FINAL VALUE?

If you respond with 1,1,5 ENTER , the table of X, X^2, X^3, X^4 gives the
below results:

```
1    1     1      1
2    4     8     16
3    9    27     81
4   16    64    256
5   25   125    625

END OF TABLE
READY
>__
```

If you type RUN ENTER again, the computer will again respond with

TYPE BEGINNING VALUE, STEP SIZE, FINAL VALUE?

and if you type 1,.5,8 ENTER , the screen will fill up and "scroll out
the top" displaying

```
1              1              1              1
1.5            2.25           3.375          5.Ø625
2              4              8              16
2.5            6.25           15.625         39.Ø625
3              9              27             81
3.5            12.25          42.875         15Ø.Ø63
4              16             64             256
4.5            2Ø.25          91.125         41Ø.Ø63
5              25             125            625
5.5            3Ø.25          166.375        915.Ø63
6              36             216            1296
6.5            42.25          274.625        1785.Ø6
7              49             343            24Ø1
7.5            56.25          421.875        3164.Ø6
8              64             512            4Ø96

END OF TABLE
READY
>_
```
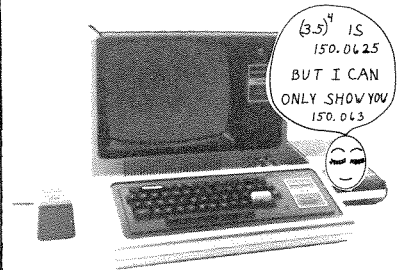
So, you see our computer works with fraction values as well as with whole numbers.

Note that $(2.5)^4 = 39.0625$ which is correct, while for $(3.5)^4$ the computer prints 150.063 instead of the correct value 150.0625. This is because, although the computer actually contains more than six digits of accuracy internally, it only <u>displays</u> six digits of accuracy. This will prove to be both a blessing and a curse, as we shall see.

If you are using a TRS-80 with Level II BASIC, it is quite feasible to obtain additional accuracy on it by declaring X and Y as DOUBLE PRECISION variables, but to do so now would only distract our learning procedure. Problems in "limited precision" and "rounding" also occur in double precision arithmetic--although not as frequently.

Let us type another set of values into our table. First, of course, depress CLEAR and type RUN ENTER. When the computer prompts you, type Ø,.1,1 ENTER. You should obtain:

```
Ø       Ø       Ø        Ø
.1      .Ø1     1E-Ø3    1 E-Ø4
.2      .Ø4     8E-Ø3    1.6E-Ø3
.3      .Ø9     .Ø27     8.1E-Ø3
.4      .16     .Ø64     .Ø256
.5      .25     .125     .Ø625
.6      .36     .216     .1296
.7      .49     .343     .24Ø1
.8      .64     .512     .4Ø96
.9      .81     .729     .6561
```

Let's modify our program slightly by adding another instruction, 1Ø6 CLS.
To do so

Depress BREAK and ENTER (if necessary) to

obtain ⎧ READY
       ⎩ >_   on your screen and then type

1Ø6 CLS ENTER

Next, depress CLEAR and type LIST ENTER to view the current program, which should start

1ØØ PRINT "TYPE BEGINNING VALUE, STEP SIZE, FINAL VALUE";
1Ø5 INPUT B,S,F
1Ø6 CLS
11Ø X=B
. . .
. . .
. . .

with the new instruction 1Ø6 tucked properly between instruction 1Ø5 and 11Ø.
Isn't that neat?

The instruction CLS is called <u>clear screen</u>. It clears the screen for you whenever it is executed. Try several B, S, F values of your own choice. Note that if you use B, S, F = Ø, .1, 5, the table scrolls off the top before you can read it. You can stop this at any time you wish.

In Level II BASIC

Hold down SHIFT and depress @ to stop the program. To continue the program, depress @ without shift (or most any other key).

In Level I BASIC

Depress and hold down the ⬆ key
To continue the program, release the ⬆ key.

<u>If you prefer</u>, depress BREAK to stop the program. Type CONT ENTER to continue.

It would be nice to have headings at the top of the column of X, x^2, x^3, x^4 we are printing. It is not hard. We add:

1Ø8 PRINT "X   SQUARE   CUBE   4-TH POWER" [ENTER]

So now our program reads:

```
1ØØ  PRINT "TYPE BEGINNING VALUE, STEP SIZE, FINAL VALUE";
1Ø5  INPUT B,S,F
1Ø6  CLS
1Ø8  PRINT "X            SQUARE            CUBE            4-TH POWER"
11Ø  X = B
12Ø  Y = X*X
13Ø  PRINT X, Y, X*Y, Y*Y
14Ø  X = X + S
15Ø  IF X<=F THEN 12Ø
16Ø  PRINT
17Ø  PRINT"   END OF TABLE"
```

Type LIST [ENTER]
to list your program and see if it agrees.

Then type RUN [ENTER]
and observe the output when you enter B, S, F = 2, 1, 9

It should be

X	SQUARE	CUBE	4-TH POWER
? 2, 1, 9			
2	4	8	16
3	9	27	81
.	.	.	.
.	.	.	.
7	49	343	24Ø1
8	64	512	4Ø96
9	81	729	6561

If your headings do not seem to be lining up properly, change the spacing inside the quotation marks in the instruction

1Ø8 PRINT "X   SQUARE   CUBE   4-TH POWER"

until it pleases you.

Note that the instruction

15Ø IF X <= F THEN 12Ø

makes a decision by comparing the values of X and F. (IF X <= F THEN 12Ø) The IF instruction produces a branching in the path of the instructions, and takes the branch path (loop) instead of continuing on if the given condition is satisfied. This is indeed a powerful idea.

Try it again with B, S, F = 2, .5, 7.

As an experiment to test your understanding, let us insert another instruction.

125   Y = Y*Y   *(Note: The 1Ø8 $\boxed{ENTER}$ deletes instruction 1Ø8, which*
1Ø8 $\boxed{ENTER}$ *would have printed erroneous headings as previously*
                      entered.)

You should forecast the output of the new program and insert new headings in instruction 1Ø8 before running the program.

1ØØ   PRINT "TYPE BEGINNING VALUE, STEP SIZE, FINAL VALUE";
1Ø5   INPUT B,S,F
1Ø6   CLS
1Ø8   *(Insert new headings here.)*
11Ø   X = B
12Ø   Y = X*X
125   Y = Y*Y
13Ø   PRINT X,Y,X*Y,Y*Y
14Ø   X = X + S
15Ø   IF X< = F THEN 12Ø
16Ø   PRINT
17Ø   PRINT" END OF TABLE"

The actual output for B,S,F = 2, .5, 1Ø is:

2	16	32	256
2.5	39.Ø625	97.6563	1525.88
3	81	243	6561
3.5	15Ø.Ø62	525.219	22518.7
4	256	1Ø24	65536
.	.	.	.
.	.	.	.
.	.	.	.
.	.	.	.

As you experiment a bit, you will find the computer occasionally switching to an odd-looking number format, perhaps

$$1.23456E+Ø8$$

This is called the "exponential" or "scientific" notation. The E+08 means to multiply 1.23456 by $1Ø^8$ (i.e., move the decimal point 8 places to the right).
Similarly 1.23456E-Ø4 would mean to multiply by $1Ø^{-4}$ (i.e., move decimal point 4 places to the <u>left</u>).

4.21653E+12 = 421653ØØØØØØØ (with a possible error of $\pm$5 in the 7th place)

7.65432E-Ø8 =.ØØØØØØØ765432 (with a possible error of $\pm$5 in the 4th place)

If you are familiar with trigonometry and have Level II BASIC, you may wish to substitute.

1Ø8 PRINT "X", "SIN(X)", "COS(X)", "EXP(X)"

13Ø PRINT X, SIN(X), COS(X), EXP(X)

There is no need to retype the entire program; just get

```
READY
>_
```

and type in

1Ø8 PRINT "X", "SIN(X)", "COS(X)", "EXP(X)"   [ENTER]
13Ø PRINT X, SIN(X), COS(X), EXP(X)   [ENTER]

Run the revised program.

Note: we obtained the spacing desired in statement 1Ø8 by enclosing each column heading in quotation marks and separating the columns with commas, just as we did in statement 13Ø which prints the values. This is easier than using just one pair of quotation marks.

SUMMARY OF LESSON 2

What have you learned in Lesson 2?
Our new program extended your knowledge of the PRINT instruction to include the possibility of printing several different values from the same instruction, and of doing computation inside of the PRINT instruction itself.

You really do have a lot of computing power at your fingertips now.

KEYBOARD COMMANDS

| ENTER | Enter line from screen to computer. |

| CLEAR | Clear the screen |

| BREAK | TRS-80, stop what you are doing and pay attention. Type CONT ENTER to continue. |

| ← | Backspace and erase. |

| SHIFT | ← | Erase entire line. |

| ↑ | (for Level I) |
| SHIFT | @ | (for Level II) | STOP! *(Release the* ↑ *key, to continue.)* *(Depress any key, without using* SHIFT *to continue.)*

NEW ENTER   Erase current program and data, clear screen and pay attention.

RUN ENTER   Run the program now in memory.

LIST ENTER   List the program now in memory.

SHIFT → to get double-sized output ; CLEAR to return to single size

BASIC INSTRUCTIONS DISCUSSED THUS FAR

X = B

Y = A*X*X + 4.6*X - 3.172

} *Computes value on right and stores it in the location named at left of equals sign.*

PRINT X,Y;Z; T,X*Y

PRINT N; "VALUES OF X TOTAL"; Y

} *Prints values of variables or expressions indicated. Also prints any message given inside of quotation marks. Semicolon is for narrow spacing, comma for wide spacing.*

INPUT "TYPE X VALUE"; X

Types message in quotes, followed by a question mark and then accepts values from keyboard and stores them in location indicated.

IF X <= F THEN 12Ø

Compares values of variables X and F. If X value is less than or equal to F value, then instruction 12Ø is executed next. Otherwise, the instruction following the IF instruction is executed next.

+   add
−   subtract
*   multiply
/   divide

As in Lesson 1, you should begin by typing in, listing, checking and running the main program under discussion in this Lesson, namely:

```
100   PRINT "TYPE BEGINNING VALUE, STEP SIZE, FINAL VALUE";
105   INPUT B,S,F
106   CLS
108   PRINT " X            SQUARE        CUBE          4-TH POWER"
110   X = B
120   Y = X*X
130   PRINT X,Y, X*Y, Y*Y
140   X = X + S
150   IF X <= F THEN 120
160   PRINT
170   PRINT "    END OF TABLE"
```

1a. Run the textbook program given above.
1b. Change instruction   140   X=X+S
                      to   140   X=X*S
     and run the program. *(Do you need to also change instruction 108?)*

2.   What happens if the program in the text is given B,S,F as 0, 2, 7 so that the step misses F = 7? Does it stop at 6 or at 8?

3.   a. Write a program of your own to create a table of values of

$$X \quad X+3 \quad (X+3)^2$$

     for X = 1, 2, 3, ... , 10

     b. Modify your program so it will accept input values B,S,F and print out
$$X \quad ,X+3 \quad , (X+3)^2$$

     for X values from B to F in steps of S.

4.   Write some programs of your own choice using the instructions we have discussed.

5.   Write a program to INPUT the radius   R, of a sphere, and print out on one line the radius, volume and surface area of the sphere using

$$\text{Volume} = \frac{4}{3}\pi R^3 \qquad\qquad \text{Surface Area} = 4\pi R^2$$

$$\textit{(where } \pi = 3.14159...)$$

30

6. Modify your program of problem 5 to INPUT values: B, S, F
   (for Beginning value, Step size and Final value) and PRINT a
   table of values of the radius, volume and surface area of spheres
   of radius B, B+S, B+2S, B+3S,...,F.

7. Write a program to convert Fahrenheit temperature to Celsius.

$$C = \frac{5}{9} \, (F - 32)$$

8. Write a program to convert temperature in degrees Celsius to
   Fahrenheit temperature.

$$F = \frac{9}{5} \, (C) + 32$$

9. Rewrite instruction 1∅8 in the program that produces powers of
   numbers, so that the headings are valid after instruction 125 Y = Y*Y
   is also inserted. Check it by using B, S, F = 2, 1, 4

10. If you have LEVEL II BASIC, try adding the following statement to the
    program that makes tables of x, x^2, x^3, x^4

        5 DEFDBL X, Y
    and RUN the program with B, S, F = 2, .5, 6

    Note that we no longer have problems with 4.5^4 = 410.0625 being
    rounded to 41∅.∅63 since we are no longer restricted to 6 decimal
    digits in our arithmetic. The   DEFDBL X,Y   instruction permits
    the TRS-80 to compute with and display up to sixteen decimal digits
    of accuracy when needed. We shall discuss this further when the
    additional accuracy is really needed. (What happens to the four
    columns of output?)

11. Write a program to create tables of values of
       x, x^3, x^6, x^9
    with proper headings for various values of B, S, F typed in by the
    user.

12. What output occurs in the program to produce a table of x, x^2, x^3, x^4
    if some or all of B, S, F are negative? Try B, S, F = 2, -.5, -3 and
    B, S, F = -2, 1, 5 and B, S, F = -1, -1, -6.

13. What happens when some of B, S, F are negative in the table of powers
    program that includes   125 Y = Y*Y ?

14a. If your BASIC includes a square root function SQR( ) try
using
120 Y = SQR(X)
130 PRINT X,Y,SQR(Y),SQR(SQR(Y))
in the text program.

b. If you don't have a SQR( ) function, try
120 Y = X*X*X
130 PRINT X,Y,X*Y,Y*Y
Be sure to alter instruction 108 to print a correct heading for the
current table columns in either (a) or (b).

15. What would happen to the program if the instruction that prints the
heading were changed to
123 PRINT"X   SQUARE   CUBE   4-TH POWER"
Try it out and see. After you understand what happened, and why,
delete instruction 123 by typing

123 [ENTER]

Notice: If mathematics frightens you, you can laugh at your fears now.
With a computer to do the arithmetic, all you need to do is
type in the formula and the computer will do the arithmetic for
you. It really is easy to do problems that used to stump you.
Try it ; you'll like it!

16. If you like mathematics, you may wish to start by proving that the
area of a rhombus (a "stepped-on" square) is given by $\frac{1}{2}CD$ where
C and D are the lengths of its diagonals. If you don't like math-
ematics, just believe us that the area of a rhombus is half the
product of its diagonals. (OK?)
    Write a program that will accept input values of C and D and
print out the message
    THE AREA OF A RHOMBUS HAVING DIAGONALS ___ AND ___ IS _____.
with the blanks filled in with the proper values. After your
program is running properly, put in some extra instructions that
will print out the message
    THE VALUES OF C AND D MUST BOTH BE POSITIVE.
if anyone uses negative or zero values as input.
Hint:
  100 INPUT "PLEASE TYPE VALUES OF DIAGONALS C,D = "; C,D
  110 IF C < 0 GOTO 500
  120 IF D < 0 GOTO 500
  130 A = .5*C*D

  150 PRINT "THE AREA OF A RHOMBUS HAVING DIAGONALS ";C;"AND";
      D;"IS";A
  160 PRINT
  170 GOTO 100

(continued on next page)

32

```
5ØØ   PRINT "THE VALUES OF C AND D MUST BOTH BE POSITIVE."
51Ø   PRINT
52Ø   PRINT
53Ø   GOTO 1ØØ
```

17. If N is the <u>number</u> of <u>sides</u> in a regular polygon and L is the <u>length</u>
of each side, then the radius of the <u>inscribed</u> circle is

$\frac{L}{2} \frac{\cos K}{\sin K}$ where K = $\frac{\pi}{N}$

                The radius of the <u>circumscribed</u> circle is

$\frac{L}{2 \sin K}$ where K = $\frac{\pi}{N}$

The area of any circle is given by πr^2 where r is its radius and
π = 3.14159... You should be able to create a program that would
list

Number of Sides / Area of Inscribed Circle / Area of Circumscribed
                                               Circle
for polygons having 3 or more sides each of length 1 unit. Do so.
Examine the output and explain what is happening. If you don't know
any trigonometry, do not let it disturb you. You can still write
this program. Use

    K = 3.14159/N
    I = .5*COS(K)/SIN(K)
    C = .5/SIN(K)
        where I stands for the radius of the inscribed circle and
        C stands for the radius of the circumscribed circle, then
        Area = πr^2.

18. A tetrahedron (triangular pyramid) has four faces each of which is an
equilateral triangle whose sides are each of length L.

    The surface area = $1.73205L^2$

    The volume = $0.11785L^3$

At L = 1, the surface area is larger than the volume.
At L = 20, the surface area is smaller than the volume.
Find the value of L that makes the number of square units in the
surface as nearly as possible equal to the number of cubic units in
the volume.

19. For what radius does a sphere have the same surface area (in square
units) as volume (in cubic units)?

    Vol = $\frac{4\pi}{3} R^3$ Surface = $4\pi R^2$

20. If (X1,Y1) and (X2,Y2) are the coordinates of two points, the dis-
tance between the two points is given by the formula

    D = $\sqrt{(X1-X2)^2 + (Y1-Y2)^2}$ Write a program to accept the coor-
dinates of two points and print out the distance between the points.
Does the formula work if some of the values are negative?

22. CHALLENGE PROBLEM: A job pays you $1 for the first 8-hour day's work, $2 for the second day's work, $4 for the next day's work, $8 for the next day's work, etc., doubling your day's wages each day, if you continue to work without being tardy or absent. It starts over at $1 any day you are late or missing.

Find how many days you will have to work, without a tardy or absence to earn a <u>total</u> of a million dollars. How much would you earn the next day, <u>if you were still on time?</u>

You may write your own program if you prefer, or you may use the program below:

```
 1Ø   PRINT "DAY#", "PAY ON THAT DAY","TOTAL PAY THUS FAR"
1ØØ   S = Ø
11Ø   D = 1
12Ø   P = 1
13Ø   S = S+P
14Ø   PRINT D,P,S
15Ø   D = D+1
16Ø   P = P+P
17Ø   GOTO 13Ø
```

After you run your program, modify it so that you can INPUT a value of T for the number of minutes tardy in step 17Ø and then send the program to 13Ø if T = 0 but to 12Ø otherwise. Use T = 480 if absent.

```
17Ø   INPUT T
18Ø   IF T = Ø   THEN 13Ø   ELSE 12Ø
```

What effect does this have?

23. CHALLENGE PROBLEM: It is easy to observe that the sum of the first four natural numbers is 1+2+3+4 = 10 or that the sum of the first seven integers is 1+2+3+4+5+6+7 = 28. How many successive natural numbers beginning with 1 would you have to add together to reach or just exceed 1000?

24. CHALLENGE PROBLEM: Bob drops a superball from a 4th floor window 42 feet above the parking lot below. If on each bounce the ball ascends to half the height it fell on that bounce, how far will the ball travel (both up and down) when it hits the ground for the 10th time? The 50th time? The 100th time?

25. Display all the perfect squares, $S = N*N$, where N and S are positive integers (whole numbers) such that each S contains <u>exactly</u> <u>six</u> <u>digits</u>.

LESSON 3
Computer-Assisted Solutions

All right—it is now time to learn to do something you can't find in a set of tables and can't do without a computer—at least not conveniently.

Now, let's introduce a new instruction by reprogramming a "Table Maker" of lesson 2.

A very useful instruction is the FOR...NEXT instruction. Try the following program on your computer.

```
1ØØ  FOR K = 3 TO 1Ø
11Ø     S = K*K
12Ø       PRINT K,S,K*S,S*S
13Ø  NEXT K
14Ø  PRINT "END OF TABLE"
```

The FOR...NEXT instruction sets K = 3 and executes whatever instructions lie between FOR and NEXT; then K is advanced to K = 4 and the instructions between FOR and NEXT are again executed; then similarly for K = 5,6,7,8,9,10. When K = 10, the instructions between FOR and NEXT are again executed with K = 10. After reaching 13Ø NEXT K with K = 10, the program continues to instruction 14Ø.

After you have run the above program, modify it to

```
8Ø  PRINT "TYPE BEGINNING VALUE, END VALUE";
9Ø  INPUT B,E
1ØØ  FOR K = B TO E
11Ø     S = K*K
12Ø       PRINT K,S,K*S,S*S
13Ø  NEXT K
14Ø  PRINT "END OF TABLE"
15Ø  PRINT
16Ø  GOTO 8Ø
```

RUN the program for several different values of B and E. Now we are ready to write a more sophisticated program.

Let's write a program to help us <u>solve messy equations</u>.

Notice that no one said the program would solve equations. It won't. But it will do all the hard work and messy arithmetic for you. All you'll need to do is to tell it where to hunt for the roots, and it will do the hunting. Does that sound fair? Let's try it. If mathematics bores you, just hang in there for now. We'll be doing computer assisted art in Lesson 4 but for now I want you to realize how easy it really is to do mathematics with the help of a computer. If you can do 6th grade arithmetic, you <u>can</u> <u>do</u> this. Keep on reading, please.

It may not have occurred to you that computer programs have to be "built." They do not spring full grown from the mind, but develop slowly—and are modified as they develop—until a final program you would be willing to share with a colleague eventually results. One of the primary rules for good programs is

> Make your program <u>run</u> first,
>
> then make it fancy.

This is good advice. Let's use it.

Our problem is to write a program to help solve the equation
$5X^3 + 3X^2 - 2X - 5 = 0$.

We shall first write a program that accepts a value of X as input, and displays the value of X along with the corresponding values of
$Y = 5X^3 + 3X^2 - 2X - 5$.
To solve the equation, we must make Y = 0 (or as close as we can get).

The following preliminary program seems to provide the required evaluation.

```
210   PRINT "TYPE X-VALUE PLEASE";   ENTER
215   INPUT X   ENTER
240   Y = 5*X*X*X + 3*X*X - 2*X - 5      ENTER
250   PRINT X,Y   ENTER
280   PRINT   ENTER
290   GOTO  210   ENTER
```

Don't forget to type LIST ENTER to check your program.

Then type RUN ENTER
The program will print

　　　TYPE X-VALUE PLEASE ?

If you type   ∅ ENTER

It will respond
　∅ -5

SHOW ME WHERE
TO COMPUTE, AND
I'LL FIND ROOTS
OF AN EQUATION
FOR YOU.
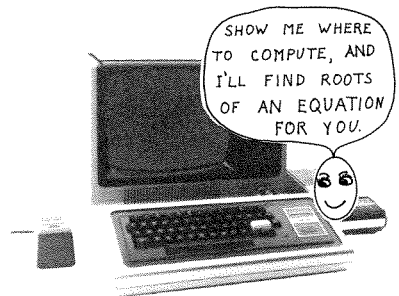
TYPE X-VALUE PLEASE?

If you type   1   [ENTER]

It will respond

    1   1

        TYPE X-VALUE PLEASE?

The change of sign in Y from –5 to +1 shows, since Y is a continuous function, that there is a root somewhere between X = $\emptyset$ and X = 1 — possibly nearer X = 1.

If we were to graph Y = $5x^3 + 3x^2 - 2x - 5$ we would know that for

X	$\emptyset$	1
Y	–5	1
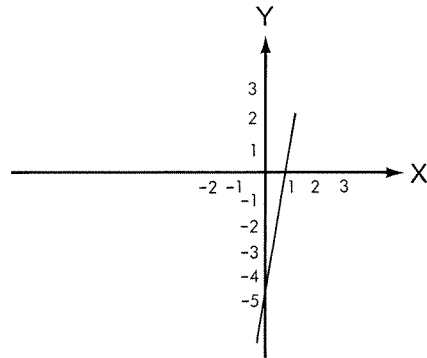
Our observation suggests that for some X-value between X = $\emptyset$ and X = 1, the value of Y must be zero. It is this X-value that corresponds to Y = $\emptyset$ that we seek.

Let's try $\emptyset$.8   [ENTER]
The computer responds

    .8   –2.12

      TYPE X-VALUE PLEASE?

So we now know there is a root between
  X = $\emptyset$.8 and X = 1   (Why?)

so we type $\emptyset$.9   [ENTER]

and obtain

    .9   –.725

so the sign change (and root) comes
  between X = .9 and X = 1

We can now enter   .95   [ENTER]
and if the corresponding Y is positive, follow this with .94 etc.
Between .944 and .945 we eventually obtain the root accurate to 5
or 6 significant digits. Try a few values yourself.
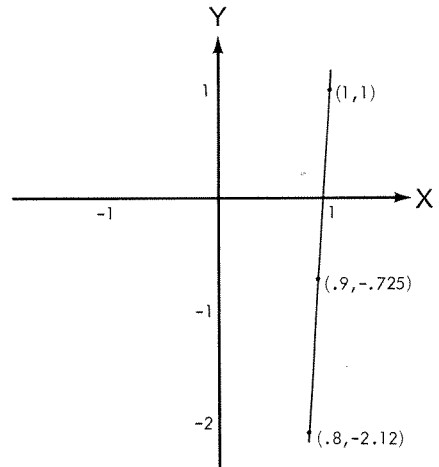
We can devise an even better helper program to solve equations. Let's modify our program so it accepts two values B and S (for Beginning value and Step size) and then prints out 11 pairs of values of X and Y before looping back to get another pair of starting values.

If you aren't sure why this will be a convenient change, just wait until you try the program given below.

We shall use our new FOR....NEXT instruction. It is really a pair of instructions

FOR K=1 TO 11

    . . . .

    . . . .

NEXT K

The instruction FOR K=1 TO 11 sets K = 1 and performs the instructions between FOR and NEXT (which may or may not involve K).

The NEXT K instruction then adds 1 to K (making K=2) and again performs the instructions between FOR and NEXT. Then 1 is again added to K, producing K=3, and the instructions between FOR and NEXT are executed again, and again, and again until finally K=11. The instructions between FOR and NEXT are performed once more, but this time whatever instruction directly <u>follows</u> the NEXT is executed.

The wee program

  100 FOR K=1 TO 20
  110   PRINT K,K*K
  120 NEXT K

will produce a table of integral values of K and K^2 for K between 1 and 20 inclusive.

The following program produces rather different results in Level I and Level II BASIC.
Try it on your computer.

  100 FOR K=0 TO 20 STEP .5
  110   PRINT K,K*K
  120 NEXT K

On Level I BASIC, it produces a table whose entries are 0s. This is because in Level I BASIC, STEP size must be an integer and STEP .5 is truncated to STEP 0.

Level II produces a table of K, K^2 for K between 0 and 20 with values changing by .5.

Try it if you are unsure. The best way to learn about your computer is to compute.

Let us return to our first version of

```
21Ø   PRINT "TYPE X-VALUE PLEASE";
215   INPUT X
24Ø   Y = 5*X*X*X + 3*X*X - 2*X - 5
25Ø   PRINT X,Y
28Ø   PRINT
29Ø   GOTO 21Ø
```

Let us combine some of the tricks we learned in Lesson 2 to produce a table of eleven values of X and $Y = 5*X*X*X + 3*X*X - 2*X - 5$ starting with X = B and increasing the X value by S on each line.

Second try:

```
21Ø   PRINT "PLEASE TYPE BEGINNING VALUE, STEP SIZE";
215   INPUT B,S
22Ø   X=B
23Ø   FOR K=1 TO 11
24Ø       Y = 5*X*X*X + 3*X*X - 2*X - 5
25Ø       PRINT X,Y
26Ø       X=X+S
27Ø   NEXT K
28Ø   PRINT
29Ø   GOTO  21Ø
```

LIST |ENTER| and check your program.

Then type   RUN | ENTER |

The program will print

    PLEASE TYPE BEGINNING VALUE, STEP SIZE?

and you type   Ø, 1 | ENTER |

The output will be:

Ø	-5 *(Note sign change!)*
1	1
2	43
3	151
4	355
5	685
6	1171
7	1843
8	2731
9	3865
1Ø	5275

Since there is a sign change between X = 0 and X = 1, you next type
0, .1 [ENTER] which produces

0	−5
.1	−5.165
.2	−5.24
.3	−5.195
.4	−5
.5	−4.625
.6	−4.04
.7	−3.215
.8	−2.12
.9	− .724999
1	1 ←——— *(Note sign change!)*

A sign change between .9 and 1.0 indicates a root there,

so you type .9, .01 [ENTER] .

Then later you type .94, .001 [ENTER] which shows a root between
.944 and .945.

.94	− .0762806
.941	− .0593705
.942	− .0424247
.943	− .0254455
.944	−8.43143E−03
.945	8.61549E−03 ←——— *(Note sign change!)*
.946	.0256987
.947	.0428152
.948	.0599666
.949	.0771518
.95	.0943718

So now we know that the equation $5x^3+3x^2-2x-5=0$ has a root between
x = .944 and x = .945. You can easily obtain 5 or 6 places of accuracy,
if needed, but 3-place accuracy is frequently enough.

To solve $7x^4 + 4x^3 + 2x^2 - 9x - 100$, you need only to change line
240 to

240   Y = 7*X*X*X*X + 4*X*X*X + 2*X*X − 9*X − 100

Starting values 0, 1 (and also 0, −1) will each be helpful in solving
this problem.

40

Computer experts will prefer to write line 24Ø as

24Ø Y = (((7*X + 4)*X + 2)*X - 9)*X - 1ØØ

which saves both typing and, more important, computing time, since it uses only four multiplications instead of 10, each time Y is evaluated. If you have had a course in algebra, please multiply out the expression to see that the same results are obtained. You can solve many different equations by changing line 24Ø.

What about solving $x^3 - 4.9x^2 + 6.6x - 2.6 = 0$ by changing line 24Ø to

24Ø Y = ((X - 4.9)*X + 6.6)*X - 2.6

Note that input B, S = Ø, 1 produces three changes in sign. Find all three roots accurate to 4 or 5 significant digits.

If you have Level II BASIC, you may wish to try $Y = e^x \cos x^2 - 2.6$

24Ø Y = EXP(X)*COS(X*X) - 2.6

This program is really quite a valuable tool. It will help you to solve almost any continuous (*e.g., no factor in the denominator that might be zero·for X inside the interval in which you are seeking a root*) function set equal to zero.

Polynomials come under this classification--even polynomials in sin(x), cos(x), e^x, as well as polynomials in X. So do many other functions, but not equations like

$$\frac{3}{2x - 1} = 0 \quad \text{since} \quad y = \frac{3}{2x - 1} \quad \text{is discontinuous at x = 1/2}$$
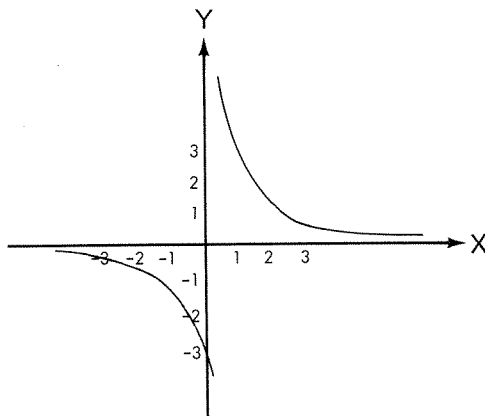
Remember: to change the equation being solved, you only need to change instruction 24Ø to

24Ø Y = (whatever continuous function you are trying to make zero).

CORRECTING ERRORS discovered at RUN time.

    You may have noticed that the TRS-80 will frequently detect errors
in spelling or punctuation of instructions and will warn you of these
errors by printing an ERROR message; consider the following erroneous
program:

    1∅∅ FOT K=1 TO 1∅
    11∅ PRINT K,K*K
    12∅ NEXT K

in which FOR is misspelled FOT in line 1∅∅. If you type

    RUN ENTER

the computer responds

    ?SN ERROR IN 1∅∅
    READY
    1∅∅ _

The first line tells you that there is an ERROR in statement 1∅∅.
We shall talk more about ERROR types later on - for now you can usually
spot the error by looking at statement 1∅∅.

For certain (but not all) errors, the computer will shift automatically
into EDIT mode. This is indicated by displaying the line number where
the ERROR has been detected just below the ERROR IN 1∅∅ message.

If you depress the L key (without ENTER), the entire line will be
displayed, and the line number repeated

    1∅∅ FOT K=1 TO 1∅
    1∅∅ _

If you tap the space bar the line will appear character by character
until the underline marks where the ERROR is:

    1∅∅ FOT K=1 TO 1∅
    1∅∅ FO_

Now depress C (which stands for change); nothing visible will happen,
but if you now depress the desired character R it will appear. If
there are other errors, correct them too. Then depress ENTER

Type LIST ENTER and you will find the correction was made.

If you should need to delete a character depress D instead of C ,
the deleted character will appear between ! ! symbols. To insert a
character depress I instead of C and then depress the desired

character. If you wish to change, delete or insert several characters
depress the number key before depressing the $\boxed{C}$, $\boxed{D}$, $\boxed{I}$ key.
Thus $\boxed{3}$ $\boxed{I}$ will enable you to insert 3 characters. This is only
part of a set of powerful EDIT instructions available in Level II BASIC.
We shall discuss them in Lesson 8 , but you may peek now if you wish.

In many cases it is <u>much</u> simpler to merely retype the line, rather
than using the EDIT instructions. In such a case if you type

RUN $\boxed{ENTER}$ and get

?SN ERROR IN 10Ø
READY
10Ø —

Simply depress $\boxed{ENTER}$

This will display line 10Ø, and display the >_ which you <u>must</u> have
to enter an instruction. Depresssing $\boxed{ENTER}$ will display

?SN ERROR IN 10Ø
READY
10Ø FOT   K=1 TO 1Ø
>_

Then you merely retype the instruction

>10Ø FOR K=1 TO 1Ø $\boxed{ENTER}$

```
        The critical things to remember are:

    The computer indicates it is ready to accept data by displaying   ?
    Do not type anything but data if you have a   ?   displayed.

    The computer indicates it is ready to accept an instruction by
    displaying >_
    Do not type an instruction unless you have a >_ displayed.

    If in doubt depress   BREAK  followed by   ENTER   to get

        READY
        >_
```

43

SUMMARY OF LESSON 3

We learned about the

    FOR K = 3 TO 1Ø

    .

    .

    .

    NEXT K

and

    FOR K = Ø TO 2Ø STEP .5

    .

    .

    .

    NEXT K

instructions, and used them to develop a powerful program that can be used to help solve continuous equations.

We also learned how to correct errors that are detected by the computer at RUN time.

Perhaps the most important thing we learned in Lesson 3 is that mathematical problems are <u>not</u> inherently difficult. Even if mathematics troubled you in the past, with the able assistance of a microcomputer <u>YOU</u> <u>CAN</u> use plain old common sense to solve problems that would cause a college mathematics major difficulty if he tried to solve them without a computer.

Note that remarks by the programmer, <u>which</u> <u>are</u> <u>not</u> printed by the program may be included by proceeding them by <u>REM</u>.

In problems 1 to 10, use the second equation solver program to help you find solutions of the problems given below.

1. $x^5 + 3x^4 + 2x^3 + 5x^2 + 3x - 25 = 0$

2. $x^5 - 31 = 0$

3. $7x^4 + 6x^3 + 10x^2 - 5x - 1 = 0$

4. $4x^3 - 8x^2 - 29x - 13 = 0$ *(Find three roots between -10 and +10.)*

5. $x^4 - 3x^2 - 4 = 0$

6. a. $x^4 - x^3 - 5x^2 - x - 6 = 0$

   b. $x^4 - x^3 - 5x^2 - x - 5 = 0$

7. $2x^4 + x^3 - 8x^2 - x + 6 = 0$

8. $6x^4 + 5x^3 - 14x^2 + x + 2 = 0$

9. $3x^3 + 4x^2 - 12x - 16 = 0$

10. $4x^4 - 13x^2 + 3 = 0$

11. a. $2x^4 - x^3 - 15x^2 + 6x + 17 = 0$

    b. $2x^5 - x^4 - 6x^3 + 3x^2 + 4x - 2 = 0$

12. Change the constant terms in some of the above equations and try again.

13. If you have a Level II BASIC, try some trigonometric equations like

$$X*SIN(X) - 2 = 0$$

$$X*EXP(X)*COS(X) + X*X - 4.5 = 0$$

The fascinating thing is you don't really need to understand trigonometry to solve the given (rather carefully selected) equations.

14. Forecast the output of the following program, then RUN it to check your forecast.

```
100   FOR  X=1 TO 10    STEP .2
110        PRINT X,X*X, X*X*X
120   NEXT X
```

Be sure to think about whether your computer has Level I BASIC or Level II BASIC.

15. Use the program

```
9Ø    INPUT "PLEASE TYPE BEGINNING VALUE, END VALUE, STEP SIZE=";B,E,S
1ØØ   FOR X = B TO E     STEP S
1Ø5      Y=X*X
11Ø      PRINT X,Y,X*Y,Y*Y
12Ø   NEXT X
13Ø   PRINT
14Ø   GOTO 9Ø
```

16. The following program is supposed to accept a positive integer value N and print out the values of N and N! = 1*2*3*...*N.
It seems to work the first time through, but then prints out nonsense values for N! Try it and see. Then debug and repair the program.

```
1ØØ   PRINT "N", "N-FACTORIAL"
11Ø   PRINT
12Ø   F=1
15Ø   INPUT "TYPE INTEGER WHOSE FACTORIAL YOU WISH" ; N
155   IF N<Ø   THEN 15Ø
16Ø   IF N=Ø   PRINT N, F
17Ø   IF N<>INT(N)   THEN 15Ø
2ØØ   FOR K = 1 TO N
21Ø      F = F*K
22Ø   NEXT K
25Ø   PRINT N, F
26Ø   GOTO 15Ø
```

These test the input data...
NOTE : INT(N) produces the integer portion of N, so
17Ø IF N <> INT(N) THEN 15Ø
looks at N and if N is not an integer, the program goes back to 15Ø. Similarly, if N is negative, instruction 155 IF N < Ø THEN 15Ø sends the program back to instruction 15Ø.

17. The following program is supposed to accept two integers N, D and determine whether or not D is a factor of N. Does it work? If so, why? If not, fix it. The heart of the program is lines 13Ø, 14Ø.

```
1ØØ   INPUT "TYPE IN TWO POSITIVE INTEGERS"; N, D
11Ø   IF N<D   THEN 5ØØ
12Ø   IF N=D   THEN 6ØØ
13Ø   Q=INT(N/D)
14Ø   IF D*Q <> N   THEN 4ØØ

2ØØ   REM  PROGRAM COMES HERE ONLY IF D IS A FACTOR OF N
21Ø   PRINT D; "IS A FACTOR OF";N
22Ø   PRINT
23Ø   GOTO 1ØØ

4ØØ REM     PROGRAM COMES HERE ONLY IF D IS NOT A FACTOR OF N
41Ø PRINT D;  "IS NOT A FACTOR OF " ; N
42Ø PRINT
43Ø GOTO 1ØØ
```

17. (continued)

```
500   PRINT N;"=N IS SMALLER THAN D=";D;"HENCE D CAN'T BE A FACTOR OF"N"
510   X=N
520   N=D
530   D=X
540   GOTO 130

600   PRINT "THE VALUES BOTH EQUAL"; N, "HENCE EACH DIVIDES THE OTHER"
610   PRINT
620   GOTO 100
```

18. In Lesson 2 you wrote programs to change temperature in Fahrenheit to temperature in Celsius and vice versa. Combine the ideas of your two programs into a single program that accepts B, S, E as Beginning value, Step size, and End value for T and then writes out two sets of tables side by side (four columns) using
PRINT T;C,T;F ←——————(Note use of ; , ; in output here.)
where T runs from B to E in steps of S and where
C = (the Celsius temperature corresponding to T degrees in Fahrenheit)
F = (the Fahrenheit temperature corresponding to T degrees Celsius)

19. Write a program to INPUT the hourly rate, R, and the number of hours, H, worked in a given week and print out the employee's gross wages for that week, assuming (s)he is paid time and a half for all time over forty hours in a given week.
The output statement might be:
PRINT "$";W;"FOR";H;"HOURS AT BASIC HOURLY RATE OF";R
but you'll have to scratch your head a bit to compute the total for W, including the overtime payments, if any. Don't make the error of subtracting ten hours of "undertime" for an employee who only works 30 hours. It does not work that way.

20. Write $X^4 - 7X^3 + 5X^2 + 3X + 71$ as a series of nested parentheses.

21. Express $(((3*X - 7)*X +5)*X +2)*X + 13$ in a form without parentheses.

22. CHALLENGE PROBLEM: The sequence 1,1,2,3,5,8,13,21,...is created by writing 1,1 and then each succeeding term is the sum of the preceding two terms, 1+1=2, 1+2=3, 2+3=5, etc.
Write a program to compute the first 100 or so terms of this "Fibonnacci sequence."

23. CHALLENGE PROBLEM: Solve $X^2 - \cos X = 0$

24. CHALLENGE PROBLEM: $4x^8 - 2x^7 + x^6 - 3x^4 + x^2 - x + 1 = 0$
presents an interesting puzzle for the mathematically-oriented. reader.

25. CHALLENGE PROBLEM: A dog is chasing a rabbit. The rabbit takes three jumps in the same length of time the dog takes two jumps, but each rabbit jump covers only half of the distance of a dog jump. The rabbit was 13 rabbit jumps ahead of the dog when the dog first spotted the rabbit and started after it. If both go in a straight line, how many more jumps will the rabbit take before the dog catches it?

26. CHALLENGE PROBLEM: How much must you invest now in an account paying 9% interest compounded quarterly to have $100,000 when you are 70 years old? (If you don't want to give your age away, figure it out for S. Marguerite who is 22 years old.)

27. CHALLENGE PROBLEM: Legend states that in 1626 Manhattan Island was purchased from the Indians for $24. If that $24 had been invested at 12% interest (a modest interest rate at that time) what would the $24 be worth today?
(How does this compare with the value of the land in Manhattan Island today?)

28. Here are two small "research" problems for you.

    CHALLENGE PROBLEM: The four-digit number 9801 has the unusual property that if you take the two-digit number formed by the first two digits, 98, and add it to the number represented by the last two digits, 01, then 98 + 01 = 99 and $(99)^2$ = 9801. Find all four-digit numbers that have this property.

29.a) CHALLENGE PROBLEM: A 4 by 4 rectangle (square) has the unusual property that its perimeter (distance around it) is the same as its area (both are 16). Your problem is to find another rectangle that also has the property that its length and width are both integers and its perimeter (2*(L+W)) is equal to its area (L*W).

   b) CHALLENGE PROBLEM: If you like mathematical thinking, see if you can prove that the two rectangles you found above are the only two such rectangles that exist.

30. CHALLENGE PROBLEM: The song "Twelve Days of Christmas" mentions various gifts that "my true love gave to me." Let's interpret the song so that "on the third day of Christmas, my true love gave to me three French hens, two turtle doves and a partridge in a pear tree." My true love gave me six presents (3 French hens, 2 turtle doves, and a partridge, if we don't count the pear tree as a gift). How many gifts in total did my true love give to me during the twelve days of Christmas? Write a program to determine the sum.

    Answer: 364, if you do not count the pear tree as a gift.

LESSON 4
Art and Graphics
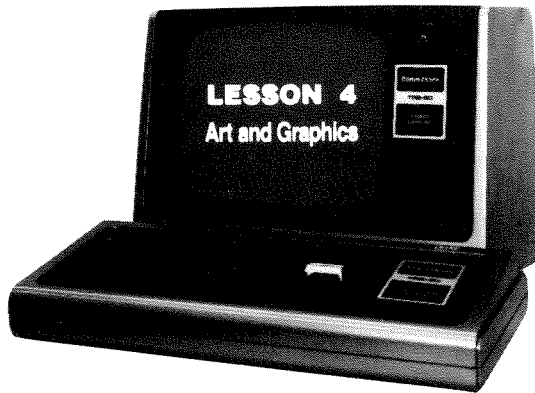
Computers are fun. They make great adult toys. As my daughter says, "The main difference between men and boys is the price of their toys." Well, a modern computer costs less than many adult toys -- a Jaguar XKE, a sailboat, a motor boat, a golf cart or even a set of clubs can cost more than a computer. Of course, the computer is a very useful device (but so is an XKE) so let's just look at how to have fun with it.

If you are curious, you may have doped out how the little surprise program of Problem 8, Lesson 1, worked. Let's look at it.

(Remember to depress ENTER *at the end of each line.)*

Level II BASIC

```
NEW       ENTER
10 PRINT @ RND(1000), "HI SUZANNE"
15 IF RND(0) < .05 THEN CLS
20 GOTO 10
RUN
```

Level I BASIC

```
NEW      ENTER
10 PRINT AT RND(1000),"HI SUZANNE"
15 IF RND(0)<.05 THEN CLS
20 GOTO 10
RUN
```

Line 10 contains two new ideas. One of these is the idea of a random number generator. TRS-80 contains two types of random number generators:

One is called RND(0). RND(0) produces a random decimal number between 0 and 1.

The other is RND(n), where n is a positive integer. RND(n) produces a random integer between 1 and n inclusive.

To see how they work, use the programs on the following page.

49

```
1000 PRINT RND(0);
1010 GOTO 1000
RUN  1000   [ENTER]  †
```

After you have a screen full of that, try

```
1000 PRINT RND(100);
1010 GOTO 1000
RUN 1000    [ENTER]
```

Later, change instruction 1000 to

1000 PRINT RND(10); (Note: this still uses instruction 1010 GOTO 1000.)

and type   RUN 1000   [ENTER]   again.

    Each time you call RND(n) a random number is produced that is differ-
ent from the random number produced last time, even though the same n
is in the parentheses.

    The second new idea in the instruction

    10 PRINT @ RND(1000), "HI SUZANNE"

is the   PRINT AT __ , "_____" instruction.

For purposes of the   PRINT AT __ , "_____" the screen is divided
into 1024 cells, sixteen rows of 64 elements each. Each cell exactly
fits one letter plus the space between rows for that letter. The cells
are numbered from 0 to 63 in the first row, 64 to 127 in the second
row, etc. (See illustration on the following page.)

RND (0) TELLS ME TO GIVE YOU DIFFERENT RANDOM DECIMALS LIKE .739682, .109654, OR .512703.

RND0

---
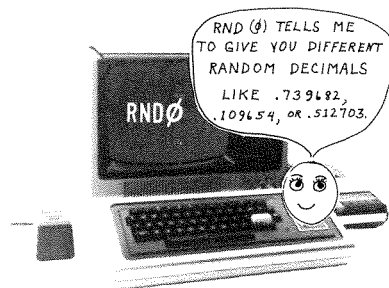
† _This uses   RUN 1000   [ENTER]   in place of   RUN [ENTER]   to make the
computer skip any program in   10, 15, 20,...,999, and start RUNning at
statement 1000. A neat trick._

Try the program

```
1000 INPUT N
1005 CLS
1010 PRINT  @  N, "* MARKS LOCATION";  N;
1020 GOTO 1000
RUN 1000
```

(Use AT in place of @ on Level I BASIC.)

The instruction

$$\text{PRINT } \binom{\text{AT}}{@} \ n \qquad\qquad \textit{Use } \begin{cases} \textit{AT in Level I BASIC.} \\ \textit{@ in Level II BASIC.} \end{cases}$$

will start printing at location n and print whatever is called for. It prints any symbols it finds inside of quotes as symbols and the values of any variables it finds outside of quotes, providing the proper commas and/or semicolons are included.

For example, instruction 10 of the program in Problem 8 of Lesson 1

$$10 \text{ PRINT} \binom{@}{\text{AT}} \ \text{RND}(1000), \text{ "HI SUZANNE"}$$

combines these two instructions. RND(1000) generates a random integer between 1 and 1000.

$$\text{PRINT } \binom{@}{\text{AT}} \ \text{RND}(1000), \text{ "HI SUZANNE"}$$

then prints whatever is in the quotes, starting at whatever random location RND(1000) generated. The next time, a different random integer is generated. The result is the statement in quotes is flashed all over the screen.

If you wish a snow storm, try

```
NEW   ENTER
1000 PRINT (@/AT) RND(1020), "*";
1020 GOTO 1000
```

To keep the screen from scrolling use ; at the end of line 1000.

To occasionally wipe the screen clear, add

```
1010 IF RND(0)<.01 THEN CLS
```

The instruction RND(0) generates a random decimal number between 0 and 1. If the random number generated is less than .01 (which it will be about once in 100 numbers on the average), the "clear screen" instruction CLS is executed. Otherwise, CLS is ignored. In either case, the instruction 1020 GOTO 1000 is executed next.

Here are some interesting programs. Try them on your TRS-80.

```
NEW      ENTER
5 CLS
10 FOR L = 0 TO 960  STEP 64
15     FOR K = 1 TO 61
20         PRINT @  K + L, "HI your name"
25     NEXT K
30 NEXT L
40 GOTO 10
```

After you try that with several different names, change instruction 15 to

```
15 FOR K = 62 TO 5 STEP -1
```

and RUN it again.

Here is another FUN program. Please try it, then analyze what happens and why. The CHR$( ) instruction, discussed later in this lesson, displays the character corresponding to the value in the parenthesis.

```
NEW   ENTER
100 CLS
110 N=1
120 PRINT @ 970, " MOBILE ART FORM # "; N ;
130 FOR K = 129 TO 191
140     PRINT @  RND(959),CHR$(K) ;
150 NEXT K
160 N = N+1
170 FOR Q = 0 TO RND(N) : NEXT Q
180     IF N < 50 THEN 120 ELSE 100
```

(Note: CHR$( ) is not available in Level I BASIC.)

Your TRS-80 has much finer graphic capabilities than we have been
using in PRINT $\binom{@}{AT}$ _____, "_____". Actually, each letter-
sized cell is broken up into six smaller rectangular cells, or *pixels*,
counting the space between lines, and each cell is individually address-
able. The system of numbering from $\emptyset$ to $1\emptyset23$ is too coarse for this use.
The individual spots are addressed by giving an X, and Y-coordinate with
$\emptyset \leq X \leq 127$ and $\emptyset \leq Y \leq 47$. The $(\emptyset,\emptyset)$ point is in the <u>upper left</u> edge
of the screen. X increases to the right. Y increases downward.

X →

Y
↓
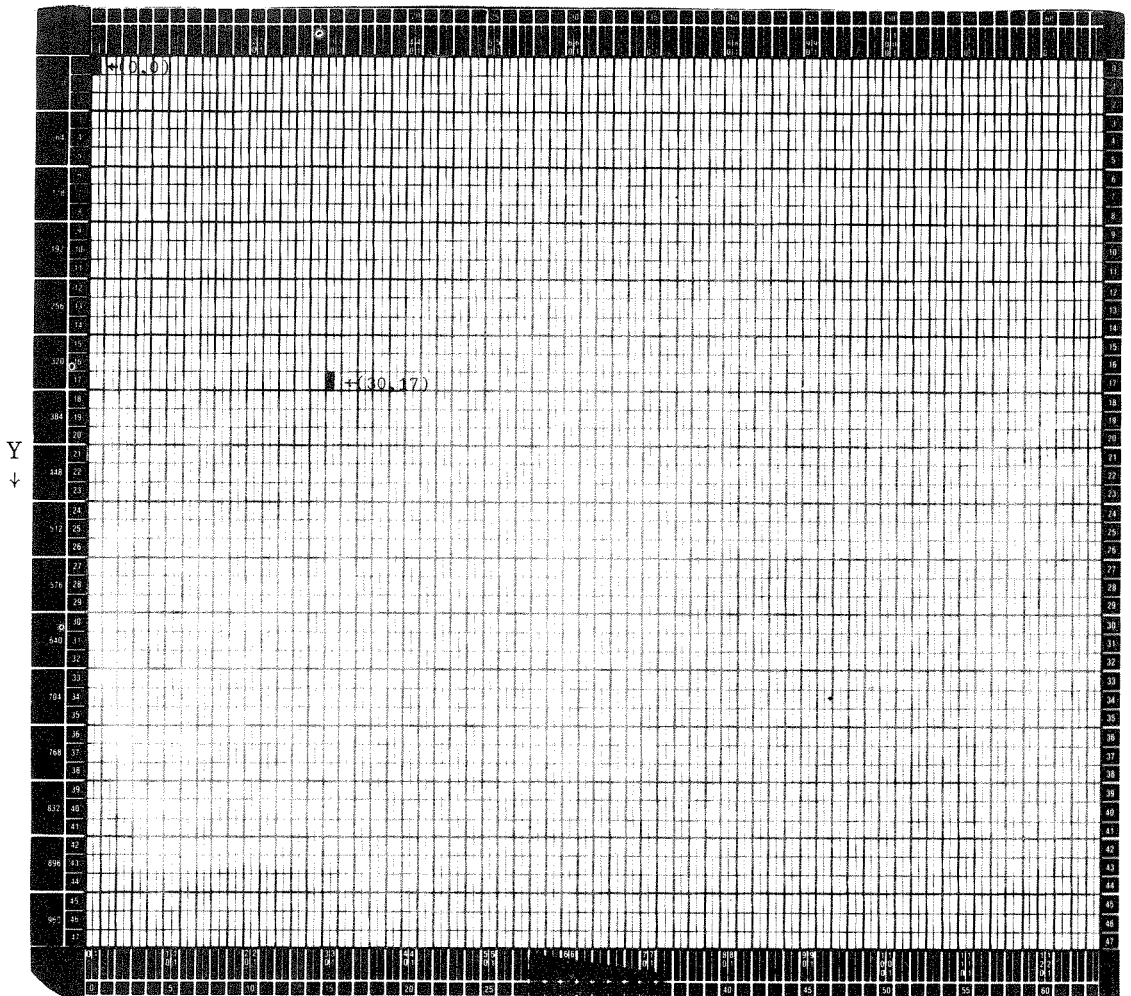
So, $(3\emptyset, 17)$ is 30 spaces to the right and 17 spaces down from $(\emptyset,\emptyset)$, as
shown above.

Try

```
1000 INPUT "X,Y=" ; X,Y
1010 SET(X,Y)
1020 GOTO 1000
RUN
```

There are three instructions used with these spots:

SET(X,Y)   This lights point (X,Y).

RESET (X,Y)   This darkens point (X,Y).

POINT(X,Y)   This is used to determine whether or not point(X,Y) is
             lighted. If lighted, it returns "logical yes", otherwise,
             "logical no", as used in an   IF....THEN   instruction.

```
100 CLS
110 FOR K = 0 TO 47
120    SET(K,K) : SET(K+60,K)
130 NEXT K
RUN
```

After you have RUN the above program, and understand why it produces
two wiggly diagonal lines, try adding

    125 SET(120-K,K) : SET(60-K,K)

Before you run the new program

```
100 CLS
110 FOR K = 0 TO 47
120    SET(K,K) : SET(K+60,K)
125    SET(120-K,K) :  SET(60-K,K)
130 NEXT K
```

see if you can forecast the pattern that will be produced. Then (and only
then) type RUN ENTER to check your forecast. This is one of the
best ways to learn about computers.

IF POINT(X,Y) THEN RESET(X,Y)   will test (X,Y) and turn it off, if it is
on. However, RESET(X,Y) also turns off point (X,Y) and takes less time.

Later, you will find uses for POINT(X,Y) in programs in which you will
need to test whether or not a given pixel is lighted.

     The following program will help you become familiar with the loca-
tion of the various (X,Y) rectangles (pixels). It lights a rectangle
chosen at random, and then permits you to input your estimate of the co-
ordinates (X,Y) of that rectangle. If you miss, a rectangle will blink
at the coordinates you chose to show you its location with respect to
the unblinking target. You may then input another guess.

```
100 R = 10 + RND(30)
110 C = 20 + RND(60)
120 CLS
130 SET(C,R)
140 INPUT "PLEASE TYPE YOUR ESTIMATE OF (X,Y) AS   X,Y ="; X,Y
150 IF(X=C)  AND (Y=R)  THEN 400

200 FOR K=1 TO 200
210    SET(X,Y)
220    RESET(X,Y)
230 NEXT K
240 GOTO 120

400 REM    HERE IF ESTIMATE IS CORRECT.
410 PRINT  @ 70,  "YOU ARE CORRECT.  CONGRATULATIONS."
420 FOR Q=1 TO 800  : NEXT Q
430 FOR Q=1 TO 50  : PRINT  @ RND(1000),  "YOU WIN"  : NEXT Q :   CLS
440 PRINT @  128,  "HERE IS ANOTHER POINT FOR YOU"
450 FOR Q=1 TO 500   :  NEXT Q
460 GOTO 100
```

It is a game...put it on your computer and play it. This is guaranteed
to improve your visualization of what is where on the pixel screen.

Here is another possibly useful technique.

Let us use our knowledge of graphics to create a bar graph or histo-
gram for the number of students attending The University of Oklahoma.

```
100 CLS
110 PRINT @ 6, "GRAPH OF ENROLLMENTS AT  THE UNIVERSITY OF OKLAHOMA"
120 REM     LABEL THE LINES FOR THE BAR GRAPHS
130 PRINT @ 128,   "1940"
140 PRINT @ 192,   "1950"
150 PRINT @ 256,   "1960"
160 PRINT @ 320,   "1970"
170 PRINT @ 384,   "1980"

200 REM     DRAWS THE ACTUAL GRAPHS
220 FOR X=12 TO 42
230    SET(X,6) :  SET(X,7)
240 NEXT X

320 FOR X=12 TO 56
330    SET(X,9) :  SET(X,10)
340 NEXT X

420 FOR X=12 TO 64
430    SET(X,12) :  SET(X,13)
440 NEXT X

520 FOR X=12 TO 102
530    SET(X,15) :  SET(X,16)
540 NEXT X

620 FOR X=12 TO  124
630    SET(X,18)  : SET(X,19)
640 NEXT X
```

A shorter and more powerful graph generator can be written using
instructions we have not yet studied (see Lesson 13). In this version, the
years and the values to be plotted are stored in DATA statements and enter-
ed into the computer using a READ instruction. (Not available in Level I.)

```
  5 CLEAR 200
 10 CLS
100 PRINT @ 6, "GRAPH OF ENROLLMENTS AT THE UNIVERSITY OF OKLAHOMA "
110 READ Y, D
115 IF Y < 0 THEN 115
120 DATA 1940, 21, 1950, 28, 1960, 32, 1970, 51, 1980, 62, -1, -1
130 PRINT Y; STRING$(D,143)
140 GOTO 110
```

However, it is better to learn to walk steadily before one tries to run or
to fly--We invite you to use the above program either with the given data
or by changing instructions 100 and 120 to produce another histogram of
more interest to you. The end of your data should be indicated by -1,-1
as above. If the data values for D lie between 1 and 59, the program will
graph the data on a single line; otherwise more than one output line may
be required in which case you may wish to insert 135 PRINT in the above
program. Experiment a bit.

Try the small-flake snowstorm programs below.

```
1ØØ SET(RND(127), RND(47))
11Ø IF RND(Ø)<.ØØ1 THEN CLS
12Ø GOTO 1ØØ
```

```
9Ø CLS
1ØØ X=RND(127)
11Ø FOR Y=1 TO 43 + RND(3)
12Ø    RESET(X,Y) :  SET(X,Y+1)
13Ø NEXT Y
14Ø GOTO 1ØØ
```

Here is another interesting program. The students claim it is a "campus planning program".

```
5Ø N=1
8Ø CLS
1ØØ PRINT  "    CAMPUS PLANNING PROGRAM "; N
19Ø FOR K=1 TO 3+RND(17)
2ØØ    SX=RND(7Ø)
2Ø5    SY=3+RND(23)
21Ø    L=RND(55)
215    W=RND(2Ø)
22Ø    FOR X=SX TO SX+L
225       SET(X,SY) :  SET(X,SY+W)
23Ø    NEXT X
235    FOR Y=SY TO SY+W
24Ø       SET(SX,Y) :  SET(SX+L,Y)
245    NEXT Y
25Ø    FOR Q=1 TO 1ØØ+RND(3ØØ) :  NEXT Q
255 NEXT K
26Ø FOR Q=1 TO 8ØØ  :  NEXT Q
265 N=N+1
27Ø GOTO 8Ø
```

If you'd like to have your N-S walls the same width as the E-W walls, add:
243 SET(SX+1,Y):SET(SX+L+1,Y)

If you want a longer look at a special design on the screen, depress SHIFT @ *to halt the program. Then depress any key, without* SHIFT *to continue.*

Note instruction 26Ø FOR Q = 1 TO 8ØØ : NEXT Q. This forces the program to count to 800 before it goes to instruction 8Ø and clears the screen. Separate instructions are separated by colons in lines 225, 24Ø, 25Ø and 26Ø.

The following program contains several instructions we have not discussed yet. If you are particularly interested in learning about them,

consult Lessons 12 and 14. However, you can <u>use</u> the programs without having to understand LEN(Z$) or MID$(Z$,K,1).

```
100 CLS
110 Z$= "YOU HAVE ALREADY LEARNED LOTS, BUT STILL HAVE OTHER GOODIES
        TO INVESTIGATE, TOO."
120 K=0
130 E=INT((LEN(Z$)+1)/2)
140 FOR A=0 TO E STEP .5
150    X=15*SIN(A)
160    K=K+1
170    PRINT  TAB(X+30) ; MID$(Z$,K,1)
180    FOR Q=1 TO 50 :   NEXT Q
190 NEXT A
200 FOR Q=1 TO 300  :  NEXT Q
210 GOTO 100
```

Run the program. Then, change line 110 to

110 Z$ = "_____"

where you type any message you wish between the quotes.

Let us write a program to create a rectangle on the video screen. Select as vertices the following points:

A(25,10)	B(45,10)
C(25,30)	D(45,30)

We might believe that the result will be a square, since the distances AB,BD,DC,CA each appear to be twenty units in length, i.e.

X-distances are each 45-25=20 units
Y-distances are each 30-10=20 units.

However, this is not valid. Let us write the program and then examine the output.

```
  10 CLS
 100 REM     DRAW LINE FROM (25,10) TO (45,10)
 110 Y=10
 120 FOR X=25 TO 45
 130    SET(X,Y)
 140 NEXT X

 200 REM     DRAW LINE FROM (25,30)  TO (45,30)
 210 Y=30
 220 FOR X=25 TO 45
 230    SET(X,Y)
 240 NEXT X

 300 REM     DRAW LINE FROM (25,10)  TO (25,30)
 310 X=25
 320 FOR Y=10 TO 30
 330    SET(X,Y)
 340 NEXT Y

 400 REM     DRAW LINE FROM (45,10)  TO (45,30)
 410 X=45
 420 FOR Y=10 TO 30
 430    SET(X,Y)
 440 NEXT Y
```

RUN the above program.

OUTPUT

60
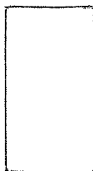
The rectangle was <u>not</u> a square. Indeed, it is about twice as tall as it is wide, since the "spots" (pixels) located at (X,Y) are actually rectangles (as you can see by typing   SET(100,30) $\boxed{\text{ENTER}}$). In other words, the X-direction units are only half as large as the Y-direction units.

To obtain a better approximation of a square, you could change each of the 45's to 65's (in instructions 120, 220 and 410).   Change the steps as suggested and RUN the program again. It still isn't a perfect square, but it is much improved. What about lines 100, 200, and 400?

It is possible to make the lines of this square of equal weight by changing instructions 330 and 430 to

```
330 SET(X,Y) :  SET(X-1,Y)
430 SET(X,Y) :  SET(X+1,Y)
```

Try these alterations and see how the new program behaves.

It still does not produce a perfect square...perhaps because the rectangular "pixels" are really not <u>exactly</u> twice as high as they are wide.

Depress $\boxed{\text{BREAK}}$ and $\boxed{\text{CLEAR}}$ and type

LIST $\boxed{\text{ENTER}}$

This will display your current program which should be:

```
10 CLS
100 REM     DRAW LINE FROM (25,10)  TO (65,10)
110 Y=10
120 FOR X=25 TO 65
130    SET(X,Y)
140 NEXT X

200 REM     DRAW LINE FROM (25,30)  TO (65,30)
210 Y=30
220 FOR X=25 TO 65
230    SET(X,Y)
240 NEXT X

300 REM     DRAW LINE FROM (25,10)  TO (25,30)
310 X=25
320 FOR Y=10 TO 30
330    SET(X,Y) :  SET(X-1,Y)
340 NEXT Y

400 REM     DRAW LINE FROM (65,10)  TO (65,30)
410 X=65
420 FOR Y=10 TO 30
430    SET(X,Y) :  SET(X+1,Y)
440 NEXT Y
```

Your task is to adjust the width of the rectangle to produce the
best square you can. Increasing the 65's in instructions 12∅, 22∅ and
41∅ slightly (say, to 75) may be all that is needed. Play with it a bit
yourself.

Actually, the program can be shortened considerably by drawing the
two parallel lines in the same FOR.....NEXT loop, as illustrated below.

```
NEW
  1∅ CLS
 1∅∅ FOR X=24 TO 73
 11∅    SET(X,1∅) :  SET(X,3∅)
 12∅ NEXT X

 2∅∅ FOR Y=11 TO 29
 21∅    SET(24,Y) :  SET(25,Y)
 22∅    SET(72,Y) :  SET(73,Y)
 23∅ NEXT Y
 25∅ PRINT  @ 4∅5, "SQUARE" ;
 3∅∅ GOTO 3∅∅
```

Instruction 25∅ prints the word SQUARE in the drawn square. The
; at the end of instruction 25∅ prevents that instruction from clearing
the following line and thus disrupting your square.

Instruction

   3∅∅ GOTO 3∅∅

puts the program in a "tight loop" that prevents the TRS-80 from displaying

```
READY
>_
```
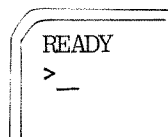
which could disrupt the display. To get the computer out of this tight-
loop, depress the   [BREAK]   key.

Each screen character or action (including scroll, space, backspace, carriage return, and convert to double-size letters) has a numeric code that corresponds to it.

In Level II BASIC (but not in Level I) it is possible to use the instruction CHR$(*n*) (where *n* is an integer or a variable that has value between 0 and 255) to obtain the screen character or action corresponding to the number *n*.

```
100 INPUT K
110 PRINT K;
130 PRINT CHR$(K)
200 GOTO 100
```

will permit you to test this out.

Use the following input values, among others, to obtain a variety of symbols: (type and ENTER them one at a time.)

33, 36, 42, 60, 88, 91, 92, 109, 134, 148, 191

The following program will print the character number along with a row of that particular character. This may be useful as a border on a graphic display.

```
10 CLS
90 PRINT     "PLEASE TYPE A NUMBER BETWEEN 33 AND 191"
100 INPUT K
110 PRINT K,
120 FOR L=1 TO 45
130    PRINT CHR$(K);
140 NEXT L
150 PRINT
200 GOTO 100
```

Try 33, 36, 38, 42, 47, 60, 62, 72, 88, 91, 92, 109, 125, 126, 134, 135, 141, 148, 152, 187 and 191 to obtain a variety.

To see the entire array of useful display symbols, change instruction 100 and add instruction 170 as suggested below:

```
100 FOR K=33 TO 191
170 NEXT K
```

After you have run the program a few cycles, you may wish to permit K to range from 0 to 255 in instruction 100. But the effect of CHR$( ) values outside of the range 33 to 191 is harder to visualize from the

program.   For example: CHR$(8)   backspaces and erases character.

The effect of CHR$(W) for several W-values is given below:

W-value	Effect or Character Printed
8	Backspaces and erases one character
10–13	Linefeed / Carriage Return
14	Turns on Cursor
15	Turns off Cursor
23	Converts to double-size letters

WANT TO SHOW YOUR PROGRAM TO A CROWD? PRINT CHR$(23) WILL MAKE OUTPUT TWICE AS BIG.
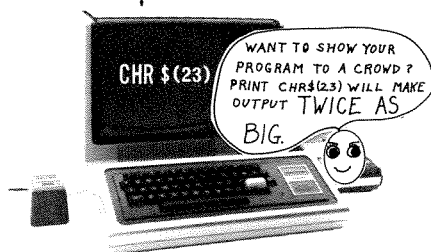
CHR $(23)

This is handy in your programs! Just include

1Ø8 PRINT CHR$(23)

and your output will be double-sized until the next CLS.

24	Backspace Cursor (←)
25	Advance Cursor (→)
26	Downward Cursor (Linefeed or ↓)
27	Upward Cursor (Linefeed or ↑)
28	"HOME", Returns Cursor to (Ø,Ø) position
29	Moves Cursor to beginning of Current Line
30	Erase to End of Line
31	Clear to End of Frame
32	Space
33	!
34	"
35	#
36	$
37	%

W-value	Effect or Character	W-value	Effect or Character
38	&	95	—— (Underscore)
39	'	96–127	Lowercase a–z (but TRS–80 does not display lowercase)
40	(		
41	)	128	Space
42	*	129–191	Various combinations of Rectangles for fast SET()
43	+		
44	,	192–255	Tabs for $\emptyset$ to 63 spaces
45	-		
46	.		
47	/		
48	0		
49	1		
50	2		
51	3		
52	4		
53	5		
54	6		
55	7		
56	8		
57	9		
58	:		
59	;		
60	<		
61	=		
62	>		
63	?		
64	@		
65–90	A – Z		
91	↑		
92	↓		
93	←		
94	→		

Just so you won't feel
this chapter is entirely friv-
olous, let's write a program
to graph the function

$y = 20+20$ sinx for $\emptyset \leq x \leq 3.2$
(radians, of course). If
you don't know what that
means, don't let it bother
you. The point is that the
computer knows what it means.
If you just realize that for each value of x between $\emptyset$ and 3.2 that you
think of, the computer will supply you with a corresponding value of
$y = 20+20$ sinx, that's all you need to know. It doesn't really make any
difference that sinx is frequently used in trigonometry and you don't
know any trigonometry...just hang on and have faith.

Let's write a short program to see that the computer really can supply the
desired value.

Level II BASIC

```
 9Ø PRINT "X", "Y"
1ØØ FOR X=Ø TO 3.2 STEP Ø.1
11Ø    Y=2Ø+2Ø*SIN(X)
12Ø    PRINT X,Y
13Ø NEXT X
RUN
```

Your computer should produce:

X	Y
Ø	2Ø
.1	21.9967
.2	23.9734
.3	25.91Ø4
.4	27.7884
.5	29.5885
...	....

Now, what we want to do is to graph this function instead of making a table of values.

Recall that the X,Y coordinates on the screen are $(\emptyset,\emptyset)$ at the top left and Y increases downward. This is inverted from the usual graph.

We shall create a new variable $Y1 = 40 - Y$ so that

Y	Y1
$\emptyset$	$4\emptyset$
$2\emptyset$	$2\emptyset$
$4\emptyset$	$\emptyset$
. . .	. . .

This will invert our graph. (Think about that a bit if it bothers you.)

We'll also introduce a new variable $X1 = 10*X$ to spread the function out on the screen. The program

```
9Ø CLS
1ØØ FOR X=Ø TO 12.5 STEP Ø.1
12Ø    Y=2Ø+2Ø*SIN(X)
13Ø    X1=1Ø*X
14Ø    Y1=4Ø-Y
15Ø    SET(X1,Y1)
16Ø NEXT X
17Ø GOTO 17Ø
```

will produce an acceptable graph of the function
$$y = 20 + 20 * SIN(x)$$

The last instruction   170 GOTO 170   is inserted to keep the TRS–80 from displaying

which would spoil the graph. To get out of this "tight loop", depress   [BREAK] .

Later, you can learn to make even fancier graphs that show the axes and units as well as the shape, but here we only introduce the basic idea.

The following program displays a series of interesting graphs.

```
100 CLS
110 A=1+RND( 5 )
120 B=1+RND( 5 )
130 C=1+RND( 5 )
140 PRINT A,B,C
150 FOR X=0 TO 12.7 STEP .1
160     Y=25-5*(SIN(A*X) - SIN(B*X) + COS(C*X))
170     SET( 10*X,Y)
175     FOR Q=1 TO 20 : NEXT Q
180 NEXT  X
190 FOR Q = 1 TO 500 : NEXT Q
200 GOTO 100
```

```
10 REM    ANOTHER "RANDOM ART"  PROGRAM
90 CLS
95 FOR L=1 TO 30
100     FOR K=0 TO 15
105       P=K*64 + RND(64) -1
110       PRINT @  P, CHR$(128+RND(63));
120     NEXT K
125 NEXT L
130 FOR Q=1 TO 900  :  NEXT Q
140 GOTO 90
```

After you have RUN the program a few times, add

115 PRINT @ RND(1023), CHR$(128+RND(63));

and RUN it again.

A ninth-grade student, Anthony Tipton, from Millwood High School, Oklahoma City, Oklahoma, produced the following rather interesting program after reading a preliminary version of this chapter.

```
3 REM     ANTHONY D. TIPTON
5 REM     1717 NE 50TH
7 REM     OKC,OK  73111
9 REM     427-0137
99 CLS
100 FOR Z=1 TO 3  :  CLS
101    FOR X=0 TO 127
102       Y=23
103       SET(X,Y)
104    NEXT X
105    FOR Y=0 TO 47
106       X=63
107       SET(X,Y)
108    NEXT Y
170    FOR X= -6.2 TO 6.2 STEP .05
175       ON Z GOSUB 300, 400, 500
180       Y= -Y
190       X1=X*10  :  Y1=Y*5
200       A=X1+63  :  B=Y1+23
205       IF B<0 THEN 220
206       IF B>47 THEN 220
207       IF A>127 THEN 220
208       IF A<0 THEN 220
210       SET(A,B)
220    NEXT X
230 NEXT Z
250 GOTO 99

300 Y=SIN(X)
301 PRINT @   896,  "SINE";
302 RETURN

400 Y=TAN(X)
401 PRINT @   896,  "TANGENT";
402 RETURN

500 Y=X*X -4
501 PRINT @   896,  "PARABOLA";
502 RETURN
```

Sophisticated readers will see ways to speed up the program. However, it serves rather nicely at its current speed as a classroom demonstration illustrating how flat the sine curve is at max and min points, and how nearly linear where it crosses the x-axis, allowing for instructor comment on each curve.

Let's see what you have learned thus far...

RND(∅) Produces a random decimal value between 0 and 1, not including endpoints.

RND(*n*) Produces a random integer between 1 and *n*, including endpoints.

PRINT @ 96∅, "NEXT VALUE IS"; N Prints at the bottom of the screen (location 96∅).

RUN 1400 |ENTER| Begins running a program at statement 1400.

SET(X,Y) Turns on rectangle at point (X,Y).

RESET(X,Y) Turns off rectangle at point (X,Y).

POINT(X,Y) Tests point (X,Y) to see if it is lighted or not (very useful in some games).

FOR Q = 1 TO 4∅∅ : NEXT Q The : separates different instructions on the same line. This particular line is a "time waster" to slow down the computer.

PRINT CHR$(*n*) Obtains the character or screen action designated by *n*.

ON N GOTO a, b, c,...,z When N takes on the value *k*, then program branches to *k*th statement number in the list a, b, c,...,z. (See Problem 11, Lesson 4.)

Mostly, we have learned to use some different "graphic" capabilities of the computer for fun and for work.

1. If you haven't tried a variation of the PRINT AT RND(1ØØØ),"HI____"
   program suggested, do so now. That is how you increase your under-
   standing of the BASIC language you are using. Try this one, too.

```
NEW   ENTER
5ØØ X=RND(125)
51Ø Y=RND(47)
52Ø SET(X,Y) :  SET(127-X,Y)
53Ø IF RND(Ø)<.ØØ2 THEN CLS
54Ø GOTO 5ØØ
```

2. Try the "snowflake" program given in the text.

```
 9Ø CLS
1ØØ X=RND(127)
11Ø FOR Y=1 TO 43+RND(3)
12Ø    RESET(X,Y) :  SET(X,Y+1)
13Ø NEXT Y
14Ø GOTO 1ØØ
```

   Then add

```
1Ø5 T=RND(127)
125 RESET(T,Y-1) :  SET(T,Y)
```

   Type   LIST ENTER to see the new program.
   Then, type   RUN   ENTER   to run it.

3. Add a third snowflake to the extended program of Problem 2 by adding
   appropriate instructions in 1Ø6 and 126. Try your program out.

4. Run the following program:

```
NEW   ENTER
1ØØ X=Ø
11Ø Y=Ø
12Ø SET(X,Y) :  SET(127-X,Y)
13Ø X=X+1
14Ø Y=Y+1
17Ø GOTO 12Ø
```

   If you are using Level I BASIC, it should fill the screen with dia-
   monds.
   If you are using Level II BASIC, the program will "bomb off" whenever
   Y>47 or X>127. This can be avoided by inserting:

```
15Ø IF Y > 45 THEN Y=RND(1Ø)
16Ø IF X > 125 THEN X=RND(15)
```
                                            Try it again!

5. Try the following program on your TRS-80.

```
NEW    [ENTER]
 90 CLS
100 X=4+RND(120)
110 Y=3+RND(40)
120 SET(X+1,Y+2) :  SET(X,Y+2) :  SET(X+1,Y+1) : SET(X,Y+1)
130 SET(X-1,Y) :  SET(X-2,Y)
140 SET(X+2,Y) :  SET(X+3,Y)
145 SET(X,Y-1) :  SET(X+1,Y-1)
160 IF RND(0)<.02 THEN CLS
200 GOTO 100
```

> Note: The programs of Problems 5 to 13 are somewhat related and may be placed on the TRS-80 without using NEW [ENTER] to save re-typing.

After it runs, try changing 160:

```
160 IF RND(0)<.2 THEN CLS
```

and then to

```
160 IF RND(0)<.005 THEN CLS
```

6. Extend and modify the program of Problem 5 by adding:

```
170 IF RND(0)<.7 THEN 100
200 X=0
210 Y=0
220 SET(X,Y) :  SET(127-X,Y)
230 X=X+1
235 IF X>120 THEN X=RND(6)
240 Y=Y+1
245 IF Y>45 THEN Y=RND(5)
290 IF RND(0)<.01 THEN 90
300 GOTO 220
```

> Type RUN 200 [ENTER]. This will run the program from 200 on, to check it out. After it is checked out type RUN [ENTER].

7. Run the following program:

```
1100 FOR X=32 TO 255
1110    PRINT X,
1120    FOR L=1 TO 45
1130       PRINT CHR$(X);
1140    NEXT L
1150    PRINT
1160 NEXT X
```

Run the program a couple of times. Then change statement 1100 to:

```
1100 FOR X=5 TO 35
```

and RUN the new program.

8. Try this program:

```
600 CLS
610 K=32+RND(150)
620 PRINT @ RND(1023), CHR$(K);
630 GOTO 610
```

After you have RUN the above program a few times, delete the semi-colon at the end of line 620 and RUN it again. Can you detect any difference? Why?

After RUNning the revised program (without the ;) add:

```
615 IF RND(0) < .01 THEN PRINT @ RND(1000), "HI your name ";
```

and RUN it.

9. Wasn't that fun! Now, try this one:

```
500 X=RND(125)
510 Y=RND(47)
520 SET(X,Y) : SET(127-X,Y) : SET(X,48-Y) : SET(127-X,48-Y)
530 IF RND(0)<.002  THEN CLS
535 GOTO 500                    (Compare this with Problem 1.)
```

10. Here's another goodie.

FOR LEVEL II

```
700 FOR Q=1 TO 200 : NEXT Q : CLS
710 FOR X=15360 TO 16383
720    POKE X,191
730 NEXT X
760 RESET(RND(124)+1, RND(40)+2)
770 IF RND(0)<.001 THEN 700
780 GOTO 760
```

FOR LEVEL I

```
700 FOR Q=1 TO 200 : NEXT Q : CLS
710 FOR Y=0 TO 47
712    FOR X=0 TO 125 STEP 2
720       SET(X,Y) : SET(X+1,Y)
730    NEXT X
740 NEXT Y
760 RESET(RND(124)+1, RND(40)+2)
770 IF RND(0)<.001 THEN 700
780 GOTO 760
```

11. The following knits together the four separate programs which we tested in Problems 5, 6, 9, and 10. In each case, the individual programs have been modified to send control to instruction 800 every once in a while. Instruction 800 then sends control, at random, to one of the subprograms beginning at instruction 90, 100, 200, 500, 700, 200, 500, 100.

```
   5 T=.01
  10 DEFINT X,Y      (Omit this instruction on Level I BASIC.)
  90 FOR Q=1 TO 200 :   NEXT Q   :   CLS

 100 X=4+RND(120)
 110 Y=3+RND(40)
 120 SET(X+1,Y+2) :   SET(X,Y+2) :   SET(X+1,Y+1) :   SET(X,Y+1)
 130 SET(X-1,Y) :   SET(X-2,Y)
 140 SET(X+2,Y) :   SET(X+3,Y)
 145 SET(X,Y-1) :   SET(X+1,Y-1)
 160 IF RND(0)<.02 THEN CLS
 170 IF RND(0)<.7 THEN 100
 180 IF RND(0)<T THEN 800

 200 X=0
 210 Y=0
 220 SET(X,Y) :   SET(127-X,Y)
 230 X=X+1
 235 IF X>120 THEN X=RND(6)
 240 Y=Y+1
 245 IF Y>45 THEN Y=RND(5)
 295 IF RND(0)<T THEN 800
 300 GOTO 220

 500 X=RND(125)
 510 Y=RND(47)
 520 SET(X,Y) :   SET(127-X,Y) :   SET(X,48-Y) :   SET(127-X,48-Y)
 530 IF RND(0)<.002 THEN CLS
 535 IF RND(0)<T THEN 800
```

$\underline{\text{On Level I BASIC use:}}$

```
 700 FOR Q=1 TO 200  :   NEXT Q   :   CLS ┐       ┌710 FOR Y=0 TO 47
 710 FOR X=15360 TO 16383          ┐      │       │712    FOR X=0 TO 125 STEP 2
 720     POKE X,191                 }     ┤       │720       SET(X,Y) : SET(X+1,Y)
 730 NEXT X                        ┘      │       │730    NEXT X
 760 RESET(RND(124)+1, RND(40)+2)         │       └740 NEXT Y
 770 IF RND(0)<.0001 THEN 90
 775 IF RND(0)<.1*T THEN CLS   :   GOTO 800
 780 GOTO 760

 800 ON RND(8) GOTO 90,100,200,500,700,200,500,100
```

Instruction

800 ON RND(8) GOTO 90, 100, 200, 500, 700, 200, 500, 100

sends the program to the nth instruction in the list 90, 100, 200, 500, 700, 200, 500, 100, depending upon the value of RND(8) which must be one of $n = 1,2,3,4,5,6,7,8$.

12.

```
390 CLS
400 X=30+RND(65) :  Y=10+RND(28)
410 SET(X,Y) :  SET(126-X,Y)
411 SET(X,46-Y) :  SET(126-X,46-Y)
415 X=X+RND(3)-2
420 X=X+RND(3)-2
430 IF X>125 THEN X= 125
435 IF X<1 THEN X=1
440 Y=Y+RND(3)-2
445 IF Y<1 THEN Y=1
450 IF Y>45 THEN Y=45
451 IF RND(0)<.009 THEN FOR Q=1 TO 500  :  NEXT Q  : CLS : GOTO 410
453 IF RND(0)<.01 THEN 400
455 IF POINT(X,Y)  THEN RESET(X,Y) :  RESET(126-X,Y) : GOTO 415
460 GOTO 410
```

Try the above program on your TRS-80. After you have enjoyed it a
bit, undertake the following modifications. Is the design symmetric
right to left? Is it symmetric top to bottom?

Add the following instruction:

```
413 FOR Q=1 TO 200  :  NEXT Q
```

which does nothing but slow down the action by forcing the computer
to count to 200 each time it passes instruction 413. Run the above a
bit before continuing...so you can follow the action in slow motion.

Try changing the .01 in instruction 453 to .05 and see how the pat-
tern is affected.

Can you combine the result of instructions 415 and 420 into one in-
struction? Do so. (Note: Although the instructions appear identi-
cal, the values of X on which they work are not, so be careful.)

13. Modify the program of Problem 11 to also include the "Art" program of
Problem 12, or your modification thereof.

14. Write an ART program of your own.

15. Write a program using the PRINT CHR$( ) instruction in some way.

16. Write a program to display an 8 by 8 chessboard (with black and
white squares).

17. Write some programs of your own choice. You really do have the in-
structions needed for most programs.

18. Write a program that begins

    `1ØØ X=RND(5Ø) : Y=RND(2Ø) : L=RND(15)`

    and will then draw a square having upper left-hand corner (X,Y) and length of side L y-units. When your program is running well, enclose it in a FOR___NEXT loop so the program draws from 6 to 10 squares. A possible over-program might be:

    ```
    1Ø CLS
    2Ø Z=RND(5)
    3Ø FOR K=1 TO (5+Z)

    1ØØ    X=RND(5Ø) : Y=RND(2Ø) : L=RND(15)
    ```

    Your square-drawing program here.

    ```
    5ØØ NEXT K
    51Ø FOR Q=1 TO 8ØØ  :  NEXT Q
    52Ø GOTO 1Ø
    ```

19. Write a program to draw a design of your choice: stairstep, circle, tree, box, face, robot, or whatever turns you on. However, choose your design first, then try to approximate it on the computer.

20. Make a bar graph showing data of your choice for several years. Note: scale your data so the longest bar ends before X = 125.

21. Make a cartoon of a dog that wags its tail on the display screen.

22. Make a smiling clown face on the display screen. Then expand your program so the clown winks or cries or talks or something that involves picture movement.

23. Create an animated "stick person" who walks across the screen, bows, and then continues walking offscreen.

24. Investigate some commercially available program that uses animation. Some possibilities are:

    > Dancing Demon
    > Life (Conway's simulation of cell growth)
    > Android Nim
    > Star Wars
    > Chess programs

    See almost any of the magazines listed in Lesson 15 or consult your local computer store or Radio Shack.

MICRO RESEARCH PROBLEMS

In Lesson 3, the number 30 Challenge Problem asks:

"The song 'Twelve Days of Christmas" mentions various gifts that 'my true love gave to me.' Let's interpret the song so that 'on the third day of Christmas, my true love gave to me

> Three French hens
> Two turtle doves
> And a partridge in a pear tree.'

My true love gave me six presents (3 French hens, 2 turtle doves, and 1 partridge, if we don't count the pear tree as a gift). How many gifts in total did my true love give to me during the twelve days of Christmas? Write a program to determine the sum. Answer: 364, if you do not count the pear tree as a gift."

A student submitted the following program in response to Challenge Problem 30. It does not produce the correct answer because the student has violated a very fundamental programming rule. Can you find the error and correct it?

```
 5 CLS
1Ø S=Ø
2Ø FOR K= 1 TO 12
3Ø    S = S+K
4Ø    PRINT K;
5Ø IF K = 1 THEN 9Ø
6Ø    K = K-1
7Ø    GOTO 3Ø
9Ø    PRINT "    "; S
1ØØ NEXT K
11Ø PRINT "    TOTAL = "; S
```

Before you continue, see what you can do to help our slipshod student.

77

The basic principle our slipshod student violated is.

```
Do NOT change a  FOR...NEXT  variable

    inside of the FOR...NEXT loop.
```

If you look at the output from the program on the previous page

1		1
2	1	4
2	1	7
2	1	10
2	1	13
2	1	16
		etc.

since the first column in each line (separated by ; spacing) presumably contains the K values and the right most values (separated by , spacing) give the current S values, it is apparent that K is not taking on the values from 1 to 12 as might be expected in FOR K= 1 TO 12.

Indeed, the program blunder (changing the value of K inside the FOR...NEXT loop) forces K back to K=1 by the end of each loop, so the new K value, namely K+1, is always K=2.   Live and learn.

If you haven't worked on this Challenge Problem, you should before continuing. It is possible to make the "blunder" into a valid running program merely by changing instructions 2∅ and 1∅∅ and adding an instruction 25. See if you can do so before continuing.

Here is our new program, with new instructions at 2∅, 25, and 1∅∅ as indicated.

```
      5   CLS
     1∅   S=∅
►    2∅   FOR L = 1 TO 12
►    25      K = L
     3∅      S = S+K
     4∅      PRINT K;
     5∅   IF K = 1   THEN  9∅
     6∅     K = K-1
     7∅     GOTO 3∅
     9∅   PRINT "    ";S
►   1∅∅   NEXT  L
    11∅   PRINT "    TOTAL=";S
```

This produces the reasonably understandable output

```
RUN
 1    1
 2    1    4
 3    2    1    1∅
 4    3    2    1    2∅
 5    4    3    2    1    35
 6    5    4    3    2    1    56
 7    6    5    4    3    2    1    84
 8    7    6    5    4    3    2    1    12∅
 9    8    7    6    5    4    3    2    1    165
1∅    9    8    7    6    5    4    3    2    1    22∅
11   1∅    9    8    7    6    5    4    3    2    1    286
12   11   1∅    9    8    7    6    5    4    3    2    1    364

   TOTAL = 364

READY

>_
```

Mathematicians might use the Greek symbol sigma, Σ, to indicate the sum and express this problem as

$$\sum_{L=1}^{12} \left(\sum_{K=1}^{L} K \right)$$

Would the program listed on the following page achieve the same result?

```
200   SL = Ø
210   FOR L = 1 TO 12
220      SK = Ø
230      FOR K = 1 TO L
240         PRINT K;
250         SK = SK + K
260      NEXT K
270      PRINT "    "; SK
280      SL = SL + SK
290   NEXT L
300   PRINT "    TOTAL="; SL
```

The final total is the same in each case (TOTAL =364) but the intermediate sums (at the extreme right of each line) are quite different.

Hmmm? How would you change the second program to get it to mimic the output of the first program? Well, we want line

   27Ø PRINT "   "; SK   to print the value of SL instead of SK.

Let's change 27Ø to

   27Ø PRINT "   "; SL

and rerun it.

At first, that may look better, but a little more careful examination shows that this revision of the second program produces a running (cummulative) sum that is always <u>one line late</u>. Hmmm?

If we interchange instructions 27Ø and 28Ø, giving

```
2ØØ   SL = Ø
21Ø   FOR L = 1 TO 12
22Ø      SK = Ø
23Ø      FOR K = 1 TO L
24Ø         PRINT K;
25Ø         SK = SK + K
26Ø      NEXT K
27Ø      SL = SL + SK
28Ø      PRINT "      "; SL
29Ø   NEXT L
3ØØ   PRINT "    TOTAL="; SL
```


I FOLLOW YOUR INSTRUCTIONS NOT YOUR INTENTIONS

This program should now produce the same result as the program on the previous page. Does it? Yes, it does. Frankly, I prefer the program at

the top of the previous page that lists the number of gifts each day on the right edge, rather than the running total.

Actually, that problem is simple enough so that we could have computed the result without using a computer...but since a computer was readily available, it may have been easier to use the computer. Or it would have been, if we hadn't made so many foolish blunders in our program. However, correcting blunders is one way to learn to write blunder-free programs.

Let us turn our attention to some problems that are not easy to solve without a computer. The term micro-research problems seems appropriate for problems that are not to be found in most text books. The easiest way to find the answer to a micro-research problem may well be to solve it for yourself. Interestingly, you do have enough mathematical and computer programming ability, so that with the help of a TRS-80 micro computer, you can solve many problems that would have been beyond your ability and/or patience five lessons ago.

Let's get going!

Now that you can use your computer to solve a really difficult problem (obtaining roots of messy equations, Lesson 3) and have fun using the graphics capability of your TRS-80 (Lesson 4), it is time to think about some interesting micro-research problems on which you can expect to make reasonable progress using your computer. We shall walk through the solution of one quite difficult micro-research problem in detail - but you may wish to solve it yourself before reading on.

Problem:

Find perfect square integers like 5776 which have the property that they end in their square roots. $\sqrt{5776} = 76$

Admittedly, this problem is not earthshaking, but it would not be very easy to solve without a computer. Try it yourself before continuing, with or without a computer.

STEPS FOR COMPUTER-ASSISTED PROBLEM SOLVING

Successful problem solving is apt to involve several quite different stages. Here is a series of steps that is sincerely recommended.

1. Be sure you understand the problem. Try to restate it in several ways. Then try to find a general method (algorithm) for solving the problem.

2. Examine a simple special case first.

3. Then, if it seems appropriate, program a computer to examine another special case.

4. Modify your computer program to examine a different special case.

5. Modify and generalize your computer program. Include tests to be sure the program is working as expected.

6. Run the new program and examine the output.

7. Now re-examine the output. Use your common sense and mathematical acumen to see if you can devise a better (faster and/or safer) algorithm or prove a theorem that will help solve the problem, unless it is already solved.

8. Go back to step 5, if the original problem has not yet been solved to your satisfaction.

This seems rather round-about at first, but experienced computer problem-solvers have found it is a much better technique than the usual technique of the amateur, who tries to start by writing the final program, or at least with the program of steps 5 and 6 right away. This is fine on trivial problems, but on more difficult problems it is really better to start easy and work up gradually, testing each stage.

Try it on our problem.

1. **Be sure you understand the problem.** Try to restate it.

    Original Problem

    Find perfect square integers like 5776 which have the property
    that they end in their square roots. $\sqrt{5776} = 76$

    Restated Problem (same problem, but from another viewpoint)

    Find non-negative integers N such that N^2 ends in N.
    Find integers $N \geq 0$ such that their squares $S = N^2$ end in the
    digits of N.

    The first technique (algorithm) that suggests itself may well be
    just to examine $S = N^2$ for each N and determine whether or not
    $S = N^2$ ends in N. Later you may be able to devise a better
    algorithm.

2. **Examine a simple special case first.**

    In attempting to solve any problem, it is well to examine
    several special cases before plunging in.

N	$\emptyset$	1	2	3	4	5	6	7	8	9
N^2	$\emptyset$	1	4	9	16	25	36	49	64	81

    So the one-digit numbers N whose squares end in N are N = $\emptyset$,1,5,6.

3. **Write a computer program to examine another special case.**

    Let us write a program that will find the three-digit numbers
    N whose squares end in N. The last sentence displays two rather
    important problem-solving techniques:

    1. In computing, it frequently pays to rephrase a question by
        turning it around a bit.

    2. It is often helpful to examine a special case (here the 3-digit
        case) of a more general problem.

    You may wish to try writing such a program yourself before
    continuing.

    If you wish to examine the last 3 digits of a number S, then you
    need to devise a technique of finding the last 3 digits of a longer
    number. There is an easy way to do it using the INT (   ) function.
    INT (   ) produces the integer portion of the value inside of the
    parentheses. Thus, INT (34.761) = 34.

The instruction

T = S - 1ØØØ*INT (S/1ØØØ)

will produce the last three digits of S and store them as T. If this is not obvious, try it for 2 or 3 integral values of S.

For example: S = 123456
              S/1ØØØ = 123.456
              INT(S/1ØØØ) = 123
              1ØØØ*INT(S/1ØØØ) = 123ØØØ
              S - 1ØØØ*INT(S/1ØØØ) = 123456 - 123ØØØ = 456

Using this instruction, see if you can devise the desired program your-self.

     A program to examine the 3-digit numbers N such that S = N*N ends in N:

```
1ØØ  FOR N = 1Ø1 TO 999
11Ø     S = N*N
12Ø     T = S - 1ØØØ*INT(S/1ØØØ)
13Ø     IF N<>T THEN 2ØØ
14Ø     PRINT S;N
2ØØ  NEXT N
```

You may wish to run the above program before continuing.

The results are:
                141376   376
                39Ø625   625

4.   Modify your program to examine other special cases.

     Can you change the program to also obtain the 1-digit values of N whose squares end in N? — namely, Ø,1,5,6 as we saw when we worked the problem by hand?

     Of course, you can. Merely change instruction,

and   1ØØ FOR N = 1Ø1 To 999   to   1ØØ FOR N = Ø TO 9

          12Ø T = S - 1ØØØ*INT(S/1ØØØ)   12Ø T = S - 1Ø*INT(S/1Ø)

and RUN the program.
Try it yourself.

     What changes would you make in the program to obtain the 2-digit values of N such that S = N^2 ends in the digits of N?
Please do so before continuing.

84

Although our current attack is clumsy, even a clumsy attack is often better than no attack at all. Let us now try to produce a program which will do the modifications for us. This is unnecessary in this particular problem, since the modifications are easily done, but we may learn some useful tricks along the way. Examine the following program and be sure you see how it works before running it.

5. <u>Generalize your computer program to examine several cases using the same program.</u>
<u>Our new fundamental program:</u>

(where P is the integer 1, followed by as many zeros as there are digits in N)

```
 1Ø    INPUT P
1ØØ    FOR N = 1+P/1Ø TO P-1
11Ø      S=N*N
12Ø      T=S-P*INT(S/P)
13Ø      IF N<>T THEN 2ØØ
14Ø      PRINT S;N
2ØØ    NEXT N
```

The program seems to work well. If there is a way to change P from 1Ø to 1ØØ to 1ØØØ etc. as the program goes along, you could then use a slight modification of the old program. Try it yourself before continuing.

In any event, the modified program given below sounds as if it would work:

```
 1Ø    P=1Ø
1ØØ    FOR N = 1 + P/1Ø TO P-1
11Ø      S=N*N
12Ø      T=S-P*INT(S/P)
13Ø      IF N<>T THEN 2ØØ
14Ø      PRINT S;N
2ØØ    NEXT N
21Ø    P = P*1Ø
22Ø    GOTO 1ØØ
```

Try it...it works! Well, it works for a while anyway.

It really does need two more instructions

```
 9Ø    PRINT "NOW WORKING ON N<" ; P          Also change 22Ø  GOTO 9Ø.
```

This will tell you the general range of N-values on which the program is working. If it gets stuck, you at least know where. It will also give you estimates on how long it takes for each set of k-digit N values, for k = 1,2,3,.... Common sense suggests 3-digit numbers should take about 10 times as long as 2-digit numbers, since there are 10 times as many of them.

Remember to include another important check.
Any computer has numbers S so large it can-
not tell S from S+1. If a computer carries
only six digits of accuracy, for example,
and S is a 7-digit number, say
    (True S) = 7654321
The computer shows this as 7.65432E+06,
which is read $7.65432 * 10^6$ or
7654320. Now (True S+1) = 7654322.
However, if your computer stores only
six decimal digits of accuracy, then
    $S+1 = 7.654322 * 10^6$
which is chopped off (or possibly rounded)
to six significant digits, giving
    7.65432E + 06
which is precisely what you had for S.
Thus, the computer cannot distinguish
between S and S+1.
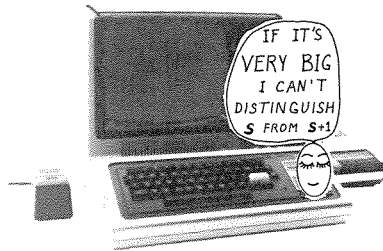
In this case, the <u>final digit</u> of S is no longer known. The computer
substitutes zero for it, and cannot tell if S ends in the same digits as
does N. Surely, you wish your program to STOP automatically if such a
crisis should arise (and it does, inevitably).

Do this by inserting the instruction

115   IF S=S+1 THEN PRINT "OVERFLOW ERROR ON S,N="; S;N : STOP

.If the computer cannot distinguish between S and S+1, it will print
the requested message and then STOP. Otherwise, the program will ignore
instruction 115 and pass on to the next instruction.

The expanded program now reads:

```
 1∅   P = 1∅
 9∅   PRINT " NOW WORKING ON N< " ; P
1∅∅      FOR N = 1 + P/1∅ TO P-1
11∅      S = N*N
115      IF S = S+1 THEN PRINT "OVERFLOW ERROR ON S,N="; S;N : STOP
12∅      T = S-P*INT(S/P)
13∅      IF N<>T THEN 2∅∅
14∅      PRINT S;N
2∅∅   NEXT N
21∅   P=P*1∅
22∅   GOTO 9∅
```

Let's return to "Steps for Computer-Assisted Problem Solving".

6. Run the general program and examine the output.

Give it a try on your own computer before continuing.

The output is:

LEVEL II BASIC	LEVEL I BASIC

NOW WORKING ON N<1Ø   NOW WORKING ON N<1Ø

| 25 | 5 | | 25 | 5 |
| 36 | 6 | | 36 | 6 |

NOW WORKING ON N<1ØØ   NOW WORKING ON N<1ØØ

| 625 | 25 | | 625 | 25 |
| 5776 | 76 | | 5776 | 76 |

NOW WORKING ON N<1ØØØ   NOW WORKING ON N<1ØØØ

| 141376 | 376 | | 141376 | 376 |
| 390625 | 625 | | 390625 | 625 |

NOW WORKING ON N<1ØØØØ   NOW WORKING ON N<1ØØØØ

OVERFLOW ERROR ON S,N = 3.35588E+Ø7   OVERFLOW ERROR ON S,N = 1.67772E+Ø7
                          5793   4096

BREAK IN 115

Notice, our program "bombed off" at P=10000 with S,N= 3.35588E+07, 5793 as a result of the test in instruction 115.   *Notice, our program "bombed off" at P=10000 with S,N = 1.67772E+07, 4096 as a result of the test in instruction 115.*

This seems to reach the limits of exploration using LEVEL I BASIC unless we are willing to construct our own special "multiple precision" routines.

It is time to consider a new concept in computing: DOUBLE PRECISION ARITHMETIC.

If you have LEVEL II BASIC on your TRS-80, you can also investigate the problem for 4-digit and 5-digit values on N (which produce 8 and 10-digit values of S) by including the instruction

$$5 \quad DEFDBL \quad S,P \qquad (\textit{which is read "Define Double".})$$

which produces double length (16-digit) values for any variable beginning with S or P. (Maybe we can even run the program for 6,7, and 8-digit N-values which have 12,14, and 16-digit squares -- wait and see.)

However, troubles appear:

The TRS-80 does <u>not</u> produce a double length product if single length numbers are multiplied together. Thus S = N*N will not produce a valid 8-digit square, if N is a single precision 4-digit number

For N = 9376   (TRUE S) = N^2 = 87909376

However, the computer rounds this to six digits and produces

$$N*N = 8.79094E + 07$$

which is stored in S as 87909400, rather than the correct value, 87909376.

The "obvious" solution is to make all three of S,P, and N double precision by using

$$5 \quad DEFDBL \quad S,P,N$$

However, the

$$FOR \ N = 1 + P/10 \ TO \ P-1$$

instruction <u>will</u> <u>not</u> <u>accept</u> <u>a</u> <u>double</u> <u>precision</u> <u>variable</u> <u>for</u> <u>N</u>.

<u>This</u> <u>is</u> <u>easily</u> <u>fixed</u> <u>by</u> <u>using</u>

```
100   N = 1+P/10                              100   FOR N = 1+P/10 TO P-1
130   IF N<>T THEN 150     in place of        130   IF N<>T THEN 200
150   N = N+1
200   IF N<P THEN 110                          200   NEXT N
```

The program now reads:

```
  5  DEFDBL S,P,N
 1∅  P = 1∅
 9∅  PRINT " NOW WORKING ON N<";P
1∅∅  N = 1 + P/1∅
11∅  S = N*N
115  IF S = S+1 THEN PRINT "OVERFLOW
     ERROR ON S,N="; S; N:STOP
12∅  T = S - P*INT(S/P)
13∅  IF N<>T THEN 15∅
14∅  PRINT S;N
15∅  N = N + 1
2∅∅  IF N<P  THEN 11∅
21∅  P = P*1∅
215  PRINT
22∅  GOTO 9∅
```

COMMENTS

  5 Omit instruction 5 if you are
     using LEVEL I BASIC.
 1∅ The number of zeros in the
     current value of P is the num-
     ber of digits in N.
 9∅ It is good practice in a pro-
     gram whose output may be scarce
     to include a statement display-
     ing what the program is doing.
115 This is a necessary precaution,
     when the computer cannot tell
     the difference between S and
     S+1, the program will STOP and
     so indicate on the screen.
21∅ Changes P to the next higher
     power of 10.
22∅ Repeats program with new P
     value.

The output is:

Level II BASIC

NOW WORKING ON N <1∅

          25   5
          36   6

NOW WORKING ON N <1∅∅

         625   25
        5776   76

NOW WORKING ON N <1∅∅∅

      141376   376
      39∅625   625

NOW WORKING ON N <1∅∅∅∅

   879∅9376   9376

NOW WORKING ON N <1∅∅∅∅∅

 8212 89∅625   9∅625

NOW WORKING ON N <1∅∅∅∅∅∅
  11963 1∅9376   1∅9376
 793212 89∅625   89∅625   etc.

Level I BASIC
(omitting instruction 5)

NOW WORKING ON N< 1∅

          25   5
          36   6

NOW WORKING ON N <1∅∅

         625   25
        5776   76

NOW WORKING ON N <1∅∅∅

      141376   376
      39∅625   625

NOW WORKING ON N <1∅∅∅∅

OVERFLOW ERROR ON S,N   1.67772E+∅7
                                4∅96

89

You will notice that for each new P value, it requires <u>ten times</u> as much <u>computer</u> time as for the previous P value.
<u>Run the program</u> again and time it.

| Level II BASIC | Level I BASIC |

Your program did not "bomb out" like Level I did. But it sure can use up lots and lots of computing time.
To get from

P=1∅∅	to P=1∅∅∅	took about 1.5 minutes
P=1∅∅∅	" P=1∅∅∅∅	"" 15 minutes
P=1∅∅∅∅	" P=1∅∅∅∅∅	"" 2.5 hours

If you let your TRS-80 run another 24 or 48 hours, from P=1∅∅∅∅∅ to P= 1∅∅∅∅∅∅, you will also find

    1196310937 6   1∅9376
    793212890625   89∅625

and who knows what else???

You were stopped during
        P=1∅∅∅∅
at which time the instruction

115 IF S=S+1 THEN PRINT "OVERFLOW ERROR ON S,N=";S;N:STOP

halted the run. You may wish to delete instruction 115 and rerun the program to see what happens.

Remember you can always stop the program by depressing   [BREAK]
You may then learn the values it was working on by typing

PRINT P,N,S,S+1   [ENTER]

NOW IS THE TIME FOR US TO GET SMART.

This program really works the problem in a very crude brute-force way. We are just examining every number, N. The number of numbers to be examined increases sharply as P increases.

	No. of cases	Approximate time
1 < P <1∅	8	Less than 1 second
1∅ < P <1∅∅	88	9 sec
1∅∅ < P <1∅∅∅	898	90 sec = 1.5 min.
1∅∅∅ < P <1∅∅∅∅	8998	15 min.
1∅∅∅∅ < P <1∅∅∅∅∅	89998	150 min = 2.5hr.
1∅∅∅∅∅ < P <1∅∅∅∅∅∅	899998	24 hr.
1∅∅∅∅∅∅ < P <1∅∅∅∅∅∅∅	8999998	9 or 10 days

Clearly we can reasonably expect to examine P < 1∅, 1∅∅, 1∅∅∅, 1∅∅∅∅, and even 1∅∅∅∅∅, but after that, the computer time involved becomes exorbitant. Of course, you may reason that the TRS-80 might just as well be working on your problem as be sitting idle. It takes very little power. (It uses about as much power as a small light bulb, if you turn off the CRT display tube, which you might as well do if you are not watching it. You can turn it on and recapture whatever would have been on the screen at that moment. The computer continues to compute even when the display tube is turned off.)

90

Returning to our "Steps for Computer-Assisted Problem Solving",

7. Re-examine the output. Use your common sense and mathematical knowledge to devise a better (faster) algorithm to help solve the problem.

We really should have recognized at the beginning that N^2 must end in 0, 1, 5, or 6 if N^2 ends in N. This would have saved almost 60% of our computing time. Actually if $N > 9$, then N cannot end in 0 since then N^2 would end in 00, etc. Similarily if $N > 9$, N cannot end in k1 since then N^2 would end in (2k) 1 since $(10k+1)^2 = 100k^2 + 20k+1$. Similar mathematical arguments show that if $N > 9$ and if N^2 ends in N, then N ends in 5 or 6. So instead of increasing N by 1 each time we can jump by 10 and test both N and (N+1) to see if either has the desired property. The following program will cut 80% of our running time--doing an hour's testing in 12 minutes.

YOU ARE CLEVER!
THIS WILL CUT
MY WORKLOAD.
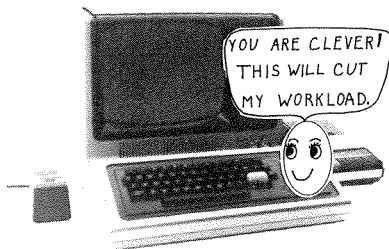
```
  5   DEFDBL S,P,N
 10   P=10
 90   PRINT "NOW WORKING ON N<";P
100   N = 5 + INT(P/100)*10
110   S = N*N
115   IF S = S + 1 THEN PRINT "OVERFLOW ERROR ON S, N =" S;N:STOP
120   T = S - P*INT(S/P)
130   IF N <>T THEN 150
140   PRINT S;N
150   N1 = N+1
152   S1 = N1*N1
154   T1 = S1 - P*INT(S1/P)
156   IF N1 <> T1 THEN 160
158   PRINT S1;N1
160   N=N+10
200   IF N <P  THEN 110
210   P = 10*P
215   PRINT
220   GOTO 90
```

Try it. It works, and we really might have been smart enough to have thought of discarding 80% of our unsuccessful cases in the beginning.

However---now that we have some output to look at, it may be possible to find an even more clever way to speed up our program. Look at the output. Do you notice anything special?

N²	N
25	5
36	6
625	25
5776	76
141376	376
39Ø625	625
879Ø9376	9376
821289Ø625	9Ø625
1196310Ø9376	1Ø9376
793212890Ø625	89Ø625

Two items demand our attention:

I.   When we investigated the one-digit N's by hand and on the first program (modified to numbers from Ø to 9), we found

N²	N
Ø	Ø
1	1
25	5
36	6

but our current computer program missed the first two values Ø, 1.
      Just why did that happen, and do you think that it missed any other values between P = 1Ø and P = 1ØØØØØØ? Are You Sure?

      We did obtain all the values between P = 1Ø and P = 1ØØØØØØ, but did miss N = Ø and 1 because this program did not test N = Ø or N = 1, since it started at N = P/1Ø + 1 for P = 1Ø which is 2.

      If you didn't notice that earlier, you need more computing experience before you can be considered a qualified amateur.

II.   Notice: The only large values of N such that N² ended in N were values of N that themselves <u>ended in those exact digits for which we found some previous satisfactory N.</u> Now that may be a real <u>clue!</u>

Consider the case starting at P=1$\emptyset\emptyset\emptyset$. We found

N^2	N
141376	376
39$\emptyset$625	625

It seems reasonable to believe that any 4-digit value of N whose square ends in N will have the value of N itself ending in either 376 or 625 since otherwise, N^2 would not end in N. Hmmm. Well, yes -- that seems reasonable.

However, some things that "seem reasonable" turn out not to be true. For example, one of my students extended the above conjecture by asserting, "Since there is only one 4-digit value of N such that N^2 ends in N, there can be only one such N having k digits for any k>4."

However,

$$N = 1\emptyset9376 \qquad N^2 = 11963 1\emptyset9376$$

$$N = 89\emptyset625 \qquad N^2 = 793212 89\emptyset625$$

blows that "reasonable conjecture".

(Why?) Well, because the student didn't recognize that the six-digit 1$\emptyset$9376 was a suitable extension of the four-digit N = 9376 under our original guess, with $\emptyset$ as the fifth digit. Hmmm.

Well, how about the original guess? If we could show that

Conjecture

If a given N^2 ends in the digits N $\geq 1\emptyset$, then N itself must end in the same digits as some smaller such N.

Not yet proved.

Would it be helpful in our search?

Well, you just bet it would be!

For example, in examining the 5-digit N values instead of examining 89998 cases, we would only need to examine the 20 cases X9376, X0625 (and possibly the 10 more cases X0376 unless we can show that is also impossible.... which we can).
I'm not going to prove the conjecture--but it is true.

To cut down from 89998 cases to 20 cases for the 5-digit N values is a savings of 99.98% of the numbers to be tested. The 6-digit values produce even greater savings. What would formerly have taken twenty-five hours should now run in less than a minute, even if each number evaluated took several times as long to test (since the program may be messier than before). That is indeed a worthwhile savings.
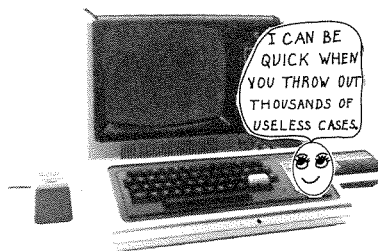
For now let us assume the slightly stronger result:

If N^2 ends in N, then N ends in the digits of some smaller K such that K^2 ends in K. Furthermore, the previous K will be a K with one fewer digit than N, with the possibility of leading zeros on a smaller K such as $9\emptyset625$ being the N for K = $\emptyset625$, $K^2 = 39\emptyset625$.

Let's start our program with 3-digit N values. This will enable us to test the program and see if it is running as we believe it should.

Store the "last two successful values of N" that we are trying to extend in locations A and B. Starting with the 3-digit values of N, there will be

    P = 1ØØØ
    A =   25   (since when N = 25, N^2 = 6<u>25</u>)
    B =   76   (since when N = 76, N^2 = 57<u>76</u>)

to properly "seed" our program with the successful 2-digit values to be extended.

Continuing with "Steps for Computer-Assisted Problem Solving," we find:

8. <u>Go back to step 5</u>.

5. <u>Modify and generalize your computer program</u>. Include tests to be sure the program is working as expected.

6. Run the new program and examine the output.

    The following program seems to produce the desired results in moderate time. Study it until you understand it well enough to explain it to one of your classmates who needs help.

```
  5   DEFDBL  S,P,N,A,B              (Omit 5 if using LEVEL I BASIC.)
 10   P=1000
 20   A=25
 30   B=76

 90   PRINT    "      WORKING ON N  <  "  ;   P
 95   Q=P/10

 99   REM      THIS BLOCK EXTENDS   A   VALUE
100   FOR K=1 TO 9
105   N = K*Q + A
110   S = N*N
115   IF S=S+1   THEN PRINT "OVERFLOW ERROR ON S,N = ";S;N:   STOP
120   T = S - P*INT(S/P)
130   IF N<>T   THEN 200
140   PRINT S;N
150   A=N
160   GOTO 300
200   NEXT K
210   REM         END OF BLOCK THAT EXTENDS   A   VALUE

250   REM

299   REM         THIS BLOCK EXTENDS   B   VALUE
300   FOR K=1 TO 9
305   N = K*Q + B
310   S = N*N
320   T = S - P*INT(S/P)
330   IF N<>T   THEN 400
340   PRINT S;N
350   B=N
360   GOTO 500
400   NEXT   K
401   REM         END OF BLOCK THAT EXTENDS   B   VALUE

450   REM

500   P = P*10
520   GOTO  90
```

Put it on your computer and see how it runs.

Output from <u>Level II BASIC</u>

```
    WORKING ON N < 1ØØØ
          39Ø625              625
          141376              376


    WORKING ON N < 1ØØØØ
         879Ø9376            9376


    WORKING ON N < 1ØØØØØ
        8212 89Ø625          9Ø625


    WORKING ON N < 1ØØØØØØ
       793212 89Ø625          89Ø625
       11963 1Ø9376          1Ø9376


    WORKING ON N < 1ØØØØØØØ
      83557128 9Ø625         289Ø625
      5Ø543227 1Ø9376        71Ø9376


    WORKING ON N < 1ØØØØØØØØ
    16616821289Ø625         1289Ø625
    7588Ø433871Ø9376        871Ø9376


    WORKING ON N < 1ØØØØØØØØØ
OVERFLOW ERROR ON S,N =1.7Ø4786682128Ø6D+17          41289Ø625


BREAK IN 115

READY
>_
```

Output from <u>Level I BASIC</u>

```
    WORKING ON N < 1ØØØ
          39Ø625              625
          141376              376


    WORKING ON N < 1ØØØØ
OVERFLOW ERROR ON S,N = 2.139Ø6E + Ø7      4625
```

Now we need to ask if the original problem has been solved.

If it was to find all perfect squares N^2 such that N^2 ends in the N, then it is only partially solved. The program found all such N < 41289Ø625 but there is no reason to suspect that is all there are. There is also the possibility our program may be defective and may have overlooked some N values less than $1Ø^9$ (After all – we did overlook Ø and 1 – remember?).

This is one way research is done. First, investigate a few special cases. Next, use a simple program to investigate a few more cases. If the results seem worthwhile, automate your program and let it run a bit. Next, examine the output, make some conjectures and prove or disprove

their validity. Then, devise a new technique (algorithm) for carrying
out the investigation and implement it.

Now you are ready to try to prove the most general case. It may or
may not yield to your effort – but in any case, you have a pocketful of
results that were previously unknown. We shall not attempt to go further
here, but strongly suspect some able student will do so.

There is one additional check that should be made.

Did our last two programs produce the same results as far as they
overlapped? If not, why not? This is important.

The two programs are:

Program from page 89	Faster program from page 95

```
  5  DEFDBL  S,P,N                 5  DEFDBL    S,P,N,A,B
 10  P = 10                       10  P = 1000
 90  PRINT "NOW WORKING ON N<";P   20  A = 25
100  N = 1 + P/10                 30  B = 76
110  S = N*N
115  IF S = S+1 THEN PRINT "OVERFLOW   90  PRINT "    WORKING ON N<";P
     ERROR ON S,N=";S;N:STOP       95  Q=P/10
120  T = S - P*INT(S/P)           99  REM  THIS BLOCK EXTENDS A VALUE
130  IF N<>T  THEN 150           100  FOR K=1 TO 9
140  PRINT S;N                   105  N = K*Q  + A
150  N = N + 1                   110  S = N*N
200  IF N<P  THEN 110            115  IF S=S+1 THEN PRINT "OVERFLOW
210  P = P*10                         ERROR ON S,N=";S;N:STOP
215  PRINT                       120  T = S - P*INT(S/P)
220  GOTO 90                     130  IF N<>T  THEN 200
                                 140  PRINT S;N
RUN                              150  A=N
                                 160  GOTO 300
                                 200  NEXT K
                                 210  REM  END OF BLOCK THAT EXTENDS
                                      A  VALUE
                                 250  REM
                                 299  REM  THIS BLOCK EXTENDS B VALUE
                                 300  FOR K=1 TO 9
                                 305  N = K*Q + B
                                 310  S = N*N
                                 320  T = S - P*INT(S/P)
                                 330  IF N<>T  THEN 400
                                 340  PRINT S;N
                                 350  B = N
                                 360  GOTO 500
                                 400  NEXT K
                                 401  REM  END OF BLOCK THAT EXTENDS
                                            B  VALUE
                                 450  REM
                                 500  P = P*10
                                 520  GOTO 90
                                 RUN
```

The two rather different programs seem to agree as far as they go, but the longer program produces more results in two minutes, than the old program did in 48 hours or even a week. That is what is meant by effective programming.

Actually all values of N such that N^2 ends in N for N of 100 or fewer digits are known, but their investigation uses more mathematical sophistication than we are willing to impose here.
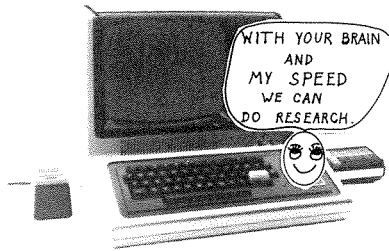
WHAT WE HAVE LEARNED IN LESSON 5.

The most important thing in Lesson 5 is the realization that you do already have enough computer adroitness to successfully undertake the investigation (and at least partial solution) of problems so difficult that a month ago you wouldn't even have tried them. You should also have discovered that it is frequently a nontrivial (but perfectly possible) task to program a computer to help solve research level problems.

Most important of all – you have discovered one of the fundamental truths of computer programming.

> When you have a program written and it is debugged
> and running properly, then if you reflect on the
> problem you will frequently find a better way to
> do it. If the problem is major, this is the time
> to think hard, then scrap your earlier effort and
> start over.

Of course if the problem is a "one time only" problem and if your existing program produces the desired results while you are thinking, you can forget the reprogramming unless you want to show the program to someone else or to use it again later.

SOME MICRO-RESEARCH PROBLEMS
YOU MAY UNDERTAKE

Don't expect all of these problems to be easy, or even solvable with your present knowledge. They are micro-research problems for your consideration. If you do more than two problems, you are to be congratulated. After working on a problem as stated, see what extensions and modifications you can devise; work on them.

1. If the 6's are cancelled in $16/64 \rightarrow 1\emptyset/\emptyset4 = 1/4$ the result is correct. ($26/65 \rightarrow 2\emptyset/\emptyset5 = 2/5$ is another example.) Find all examples of proper fractions such that AB/BC = A/C with A ≠ B where A,B,C are digits between $\emptyset$ and 9 inclusive.

2. If $N! = 1*2*3*...*(N-1)*N$ for $N \geq 1$

    and $S(K) = 1! + 2! + 3! + 4! + ... + K! = \sum_{N=1}^{K} (N!)$,

    for what values of $K \geq 1$ is S(K) a perfect square?
    (This requires some easy mathematical thinking after you have written and run a computer program.)

3. Write a program to accept an integer N > 2 as input and then type out N and the factors of N.

4. Let your input be three positive numbers A,B,C (not necessarily integers).
    a) Determine whether or not A,B,C can be the sides of a triangle. *(Note: If A,B,C = 1,5,3, the answer is "no".)*
    b) If A,B,C are acceptable as sides of a triangle, extend your program to also print the <u>area</u> of the triangle.
    c) If A,B,C do determine a triangle, extend your program to also compute the <u>angles</u> of the triangle.

5. a) Write a program that will accept a positive integer N as input and determine whether or not N is a prime. *(Note: 1 is not a prime. An integer N > 1 is prime if its only positive factors are 1 and N.)* Compare your program with those written by your classmates. Test it for several values of N > 6$\emptyset\emptyset\emptyset$. See whose runs fastest, and why.
    b) Make a table of primes < 5$\emptyset\emptyset\emptyset$.
    c) Extend your program to print out all the prime factors of N.

6. Write a program to accept two positive integers M and N and to print out M, N and also the largest positive integer which divides both M and N. Such an integer is called the greatest common divisor and may be found using a technique called Euclid's Algorithm or by simply trying all positive integers from the smaller of M and N down to 1 until you find a value which will divide both.

If you use the latter (inefficient, but effective) technique, your program could begin:

```
1ØØ   INPUT "M,N =" M,N
1Ø6   IF M<=Ø   THEN 1ØØ
1Ø8   IF N<=Ø   THEN 1ØØ
1Ø9   REM:  INSTRUCTION  11Ø SETS S = SMALLER OF (M,N) FOR USE IN 2ØØ
11Ø   IF M>N  THEN S = N ELSE S = M
2ØØ   FOR D=S TO 1  STEP -1
         .        .
         .        .
         .        .
```

7. Find two consecutive integers whose squares each use exactly the same digits (in a different order, of course). Thirteen and fourteen are such a pair; $13^2 = 169$ uses the same digits as $14^2 = 196$. Similarly, $157^2 = 24649$ and $158^2 = 24964$. Find other <u>pairs</u> <u>of</u> <u>consecutive</u> <u>inte-</u> <u>gers</u> whose squares are composed of the same digits.

8. a) Write a computer program to accept three values M,D,Y and determine whether or not they are acceptable values for Month, Day, Year in that order. If Y < 1590 you may prefer to state the date is unacceptable rather than consider the various calendar reforms that took place prior to 1590.

   b) Extend the above program to accept two sets of three numbers and if both are acceptable as Month, Date, Year values, determine the number of days between them including both dates in your count.
   Example: 2,2Ø,1978 to 2,22,1978 is 3 days.

9. For the <u>distinct</u> digits A, B, C, D, E, F, G, H, I, J the product

$$A*(BCDE) = FGHIJ$$

has solution $4*(7039) = 28156$ and <u>a</u> <u>dozen</u> <u>other</u> <u>solutions</u>. Find them.

10. Write a program to accept a positive integer K and print it out and also print out the integer obtained by reversing the digits of K.
    Example: If K = 12345,
           print out: 12345   54321

11. The sequence
              1   2   3   4   6   8   9   12   16   18   24   27   32 ...
    is made by listing numbers of the form N = $2^k \cdot 3^m$
    (with k and m integers $k \geq Ø$, $m \geq Ø$) <u>in</u> <u>order</u> <u>of</u> <u>increasing</u> <u>size</u>.
    It is not easy to devise an algorithm <u>to</u> <u>generate</u> <u>and</u> <u>print</u> <u>them</u> <u>in</u>
    <u>increasing</u> <u>order</u>, but that is only part of your task. The even more difficult problem is to find the 1ØØØth term of the sequence, or more generally, given an integer T, find the T-th term of the sequence. (Thanks to Fred Gruenberger for suggesting this problem.)

12. Write a program that will draw a random maze on the screen such that it is possible to reach every cell of the maze, but there is only one "best" path (i.e. without retracing) through the maze. Then, extend your program so that the computer will "solve" the maze it created.

13. a) Determine the sum of the N-digit numbers that can be formed using N specified digits for each possible set of 2,3,4 and 5 digits.
    Examples:

    N = 2   Digits = 3,8   Then sum = 38 + 83 = 121
            Digits = 4,1   Then sum = 14 + 41 = 55   *(Yes, we count*
            Digits = 2,2   Then sum = 22 + 22 = 44   *22 both ways.)*

    N = 3   Digits = 1,2,3   Then sum =
                    123 + 231 + 312 + 132 + 321 + 213 = 1332
            Digits = 3,8,9   Then sum =
                    389 + 893 + 938 + 983 + 839 + 389 = 444ø

   b) Notice that the sum <u>seems</u> always to be divisible by 11. Can you prove this in general? What other factors will the sum have?

   c) Can you show that an N-digit number will have N! different permutations, (N-1)! of which start with each digit, and (N-1)! of which have each digit in the second place, etc.?

   d) If you prove each of the above, you may also be able to show that the <u>sum</u> of the N! numbers obtained as permutations of the N digits <u>will</u> always be (N-1)! * (sum of the original digits) * (the integer consisting of N ones). For N = 8, Digits = 1,2,3,4,5,6,7,8 there are 4ø32ø integers and their sum is 2ø15999979840.

   e) What happens if a number containing repeated digits is counted only once? Thus N = 2, Digits = 2,2 Sum = 22 (not 22 + 22 = 44). [This is a rough problem.]

14. A palindrome is a phrase or sentence which reads the same forward as backward.

    "Too Hot To Hoot"
    "Ma Is As Selfless As I Am"
    "Retracting, I Sign It, Carter"
    "Marge Lets Norah See Sharon's Telegram"

There are also numbers which are palindromes. It is fairly easy to find perfect squares which are also palindromes.

$(111)^2 = 12321$

$(202)^2 = 40804$

$(264)^2 = 69696$

It is possible to prove that there exist infinitely many perfect squares that are palindromes. However, perfect squares that are palindromes having an <u>even number of digits</u> such as $(836)^2 = 698896$ are fairly rare.

Investigate the subject of perfect squares that are also palindromes.

15. For what values of N > 1 does $W = 2^N - 1$ contain a factor > 1 which is a perfect square?

If $N = 6$, $W = 2^6 - 1 = 63 = 3^2 \cdot 7$

If $N = 7$, $W = 2^7 - 1 = 127$ which does not contain a square factor > 1

If $N = 8$, $W = 2^8 - 1 = 255$ which does not contain a square factor > 1

16. a) It is easy to find Pythagorean rectangles having integral sides and integral diagonals $(l,w,d) = (3,4,5)$ or $(5,12,13)$. But can you find a box (rectangular parallelepiped) having three distinct integers as dimensions, each of whose face diagonals is also an integer?

    b) If so, can you also find one whose body diagonal as well as face diagonals are integers?
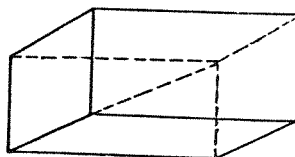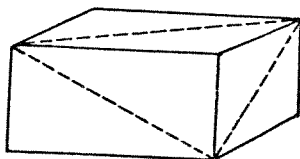
17. Use your TRS–80 to design some personal greeting or Christmas cards. Take photographs of the TRS–80 showing the design on the screen and enough of the keyboard to be recognizable. Send one to your authors.

PART II

You are ready to proceed on your own now. No one is expected to work all of the research problems in Lesson 5, but you should work several and at least read and consider the others. The way to learn computing is to compute, and then reflect.

The remainder of this book discusses useful techniques and additional BASIC instructions you may or may not need to know about. Lessons 6 to 17 may be read in any order desired, or ignored. Read about these techniques when you are ready to use them. You already have skill enough to solve many problems using the computer as a tool. We suggest you glance briefly at the summary below to ascertain the general content of the lessons beyond 5. Then read them when the need for such specialized information is felt. Read whatever interests you most, and continue with the micro-research problems presented in Lessons 5 and 17.

Lesson 6 is devoted to the use of the TAPE CASSETTE for storage of programs and data.

Lesson 7 is devoted to GAMES and TAPED PROGRAMS you may enjoy.

Lesson 8 discusses the EDIT instruction in greater detail.

Lesson 9 is an introduction to simulation, one of the most important
uses of computers in today's world.

Lesson 10 discusses double precision variables (introduced in Lesson
5) before introducing subscripted variables and the fascinating (and
vital) string variables (more details in Lesson 14). The lesson closes
with arrays (matrices) and how to use them.

Lesson 11 is a collection of tips that may help you overcome or avoid
difficulties. Remarks on saving memory space and computing time as well
as a listing of the meanings of major error codes are presented in this
lesson.

Lesson 12 Although the PRINT instructions already at your disposal
are sufficient for most users, your TRS-80 Level II BASIC has a variety of
other PRINT instructions and special commands that are frequently useful:

         PRINT TAB(   )   PRINT STRING$(K,"#")

         PRINT USING A$,K   PRINT CHR$(n)

Lesson 13 Lesson 4 may meet all your graphic needs, but Lesson 13
explains the pixel notation and introduces several time-saving techniques
and instructions.

Lesson 14 extends your knowledge of string variables, and introduces
elementary conversational programs in which the computer simulates con-
versation. The ROBOT COUNSELOR program and the CIPHER program may also
interest you.

Lesson 15 explains how and where to look for additional information
on computing with particular emphasis on the TRS-80 Level II BASIC.
Several computer related journals are listed with brief annotations.

Lesson 16 discusses several BASIC instructions not used earlier in
in this book.

Lesson 17 extends the collection of seventy-five microRESEARCH
PROBLEMS. The way to learn computing is to compute and then to examine
your programs and results to see if you could have devised a better pro-
gram.

The discussions presented Lessons 5 and 17 are planned to help you
develop efficient as well as effective programs and to whet your problem
solving ability. Many of the problems presented are readily extended.
Their solutions frequently suggest further explorations worthy of your
talent. This is why they merit the adjective "RESEARCH". The wide var-
iety of problems includes something for every taste: several arithmetic
explorations, greeting card graphics, Haiku poetry, logical decisions,
calendar problems, dart and target games, loan and finance programs, trea-

sure hunt games, caricatures, speech timer, puzzles, recursive functions, polygonal maps, a medical emergency prompter, number theory problems, lattice problems, magic prime squares, graphs, monkeys at typewriters, amicable and sociable numbers, dance partners and the eight queens problem.

THE IMPORTANT THING IS TO <u>HAVE</u> <u>FUN</u> WITH <u>YOUR</u> micro COMPUTER

LESSON 6
Tape Cassettes
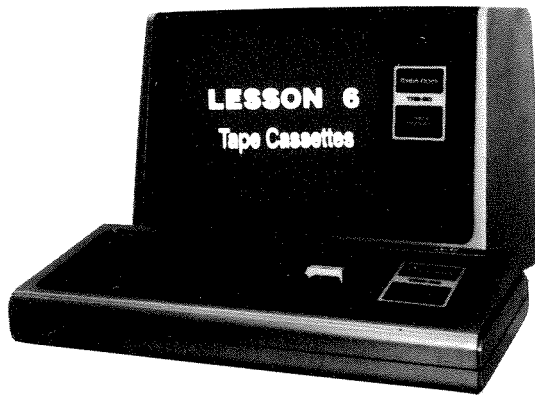
TAPE CASSETTE for programs and data storage.

Occasionally you have a special program worth saving for future use or to show off to a friend. Put it on tape. Tape is also handy for storing data.

Each TRS-80 comes with a tape recorder for storing programs and data that you wish to save. It uses regular (high quality) tape cassettes. I personally prefer the short 30 minute (15 minutes per side) tapes rather than the 45 minute, 60 minute or 90 minute tapes since the tape recorder starts, stops and operates more uniformly when the reel contains less tape. Try it and suit yourself. Radio Shack sells a special 10-minute tape for use on TRS-80 which has no "leader" on it--but any high quality 30 minute (15 minutes per side) tape will serve--just so you don't try to record on the leader.
For readers who have older CTR-40 and CTR-41 tape recorders:
     The original tape recorders supplied with TRS-80s were CTR-41's on which the REM jack had to be unplugged before the tape was repositioned. The CTR-41 also requires the use of a "dummy plug" in the MIC jack at all times. The CTR-80 now furnished should not have a dummy plug inserted at any time.

     Instead of unplugging the REM jack on the CTR-41 you can use a short program.

```
9000  OUT 255,4
9010  INPUT "REPOSITION TAPE, PRESS (ENTER) WHEN READY" ; A$
9020  END
```
Then type RUN 9000 when you wish to reposition the tape.
For readers with pre-1980 CTR-80 tape recorders:
     The more recent CTR-80's can use "fast-forward" or "rewind" without unplugging the connection between TRS-80 and the cassette at the REM jack.

107

This is much nicer.

In either case the volume control was critical in getting usable programs and data back into the TRS-80 -- on early (pre-July, 1979) TRS-80s.

The following may be helpful:

Radio Shack announced (Spring, 1979) a "fix" that they will install free to make the recording level less sensitive on existing TRS-80's. Current TRS-80's already have it installed. Your TRS-80 Level II keyboard has a catalogue number on the underside of the case. If the catalogue number ends -1 (example 26-1006-1), the modification has already been made. If not, it will be worthwhile to take the keyboard unit to Radio Shack to have the free "fix" made. Cassette storage is much easier to use thereafter.

If your CTR-80 cassette player was manufactured before February, 1979 (see date stamped on cassette box--Feb., 1979 = 2A9) take the cassette player to your Radio Shack store, too. They will install a small capacitor in the innards of the electronics to avoid a possible "noise spike" on your recorded tape if you push the stop button while it is reading tape, an infrequent but annoying happening. If the date of manufacture is Feb., 1979 or later (say 6A9 or 1A0) this extra protection has already been installed at the factory. We recommend setting modified CTR-80 cassette recording level at 4 both for recording and playback. Once you find the best volume for your system, a drop of white correction fluid (Snopake, Liquid Paper, etc.) or paint will mark the location for future reference.

In any case, the usual tape care is essential.

1. Don't leave cassettes near a magnetic flux source like your power supply or near a magnet as in a motor or a speaker. Also, don't expose cassettes to extreme heat or cold.

2. Keep your cassette capstan and rollers clean. Dirty heads cause lots of troubles. If you don't know how to clean them, your nearby Radio Shack or tape recorder store will be glad to show you, and sell you a cleaning kit (<$2).

To load a tape that already contains a program

Plug in everything carefully.

110 volt (Don't expect to use the batteries.) black power
cord

DIN plug to TRS-80 keyboard hole labeled TAPE with

pip on plug in slot on socket ,

3 plugs on other end of cable connect to cassette player

BLACK plug into EAR hole

LARGE GREY plug into AUX hole

SMALL GREY plug into small MIC hole (CTR-80)
or REM hole (CTR-41)

If you are using CTR-41 put dummy plug into MIC and leave it
there.

If you are using CTR-80 do not use dummy plug--just leave big
MIC hole empty.

NEXT:

Adjust the position counter back to zero. (They register differently on
CTR-41 and CTR-80, so be a bit suspicious.)

Check the volume adjustment. (This is fussy, experiment a bit.)

CTR-41 5 to 8

CTR-80 3.5 to 4.5 (user-generated); 4 to 5 (commercial tape)

Push PLAY key on cassette.

Type CLOAD or CLOAD "ℓ" and ENTER on Keyboard (where ℓ is letter
identifying your program).

The cassette should be turning now.

Soon the TRS-80 video screen will show two asterisks.

Level I ⟶ >** ** ⟵ Level II
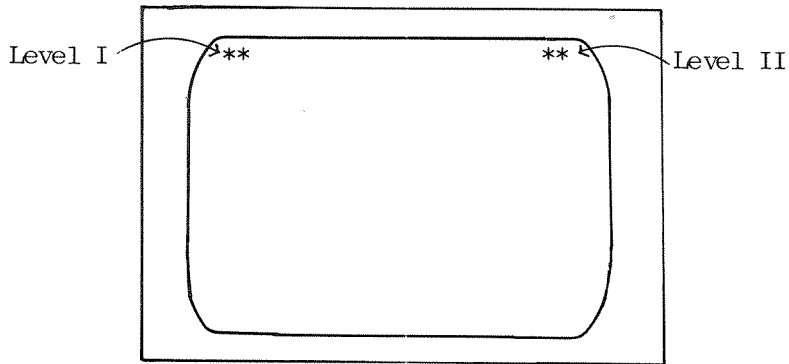
If all is going properly, one asterisk will blink irregularly.

When it finishes loading, the cassette will halt the tape and the display
screen will show

READY

>_

Depress the ⌊STOP⌋ button on the cassette (even though the tape
has halted). *Do not leave the* ⌊PLAY⌋ *key depressed when not in use.*

Type LIST ⌊ENTER⌋ .

If the program seems OK, type RUN ⌊ENTER⌋.

If no program is present (or if the asterisks appeared but did not wink)
lower the volume slightly; rewind the tape and try again.

If something loaded, but the last part seems all jumble and gibberish,
increase the volume slightly; rewind the tape and try again.

To SAVE a program currently in the computer

Set up the cassette recorder as described before; be sure plugs are set firmly in the proper holes.

Check the volume setting.

CTR 41   4-6

                    (Try mid-range first.)

CTR 80   3.5-4.5

Rewind the tape. (Unplug REM jack only if on CTR-41 or use program fix.)

Reset the tape counter to 000.

Advance the tape to get past the "leader" if you are using standard tape—or to a blank place if the tape already contains some programs or data. (I usually begin programs at 10 on the tape and leave between 10 and 19 "counts" of blank tape between programs. This may be overly cautious, but it sure makes it a lot easier to find things.)

Note the program name and where it begins on the index card with your cassette or on the cassette label.

Depress both RECORD and PLAY keys on the cassette recorder (surprise—yes both should be down to record—so the TRS-80 can control things properly).

Type CSAVE " " (with a letter to identify the program between the " ").

The tape should begin to advance and the TRS-80 will record your program on the tape.

When the program is recorded, the video screen will display as usual.

```
READY
>_
```

Depress the STOP button on your recorder. <u>This</u> <u>is</u> <u>important.</u>

Note where the program ended on your index card or the cassette label.

Rewind the tape to the beginning of the program (see index card and counter).

Depress PLAY only on cassette player.

Type CLOAD ? to check the tape against the program in the TRS-80 memory. If the tape checks properly, the screen will display READY, otherwise screen will display BAD and you should CSAVE " " it again.

111

I record each program twice, just in case anything happens. You'll have more space than you can use on a 15-minute per side tape. (Use both sides, of course.)

Sound and Music

Some TRS-80 programs include sound or music as part of their output. Higher quality musical reproduction is obtained by using a separate amplifier and speaker. If you are interested in medium fidelity sound rather than high fidelity music production, quite satisfactory results may be obtained using the amplifier present in the cassette player furnished with your TRS-80.

First get the program in your TRS-80 in the usual fashion. Then unplug the black plug from the ear hole in the cassette recorder (leaving the two grey plugs in place). Plug the earphone that came with the tape recorder into the ear hole (or better yet plug a speaker in there--you can get a suitable jackplug or converterplug from Radio Shack).

Now you need to fool the tape recorder. Remove any tape cassette from the recorder. Push in the little lever at the back left side of the opening where the cassette would go. While you are depressing it, depress both the ⌐RECORD⌐ and ⌐PLAY⌐ buttons on the front of the recorder. These should stay down when you release the lever. Now RUN your program.
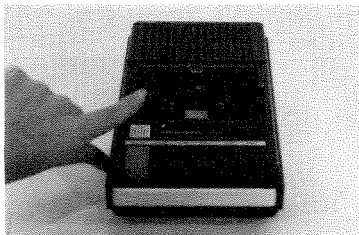
The following instructions are used to place data onto tape and to retrieve data from tape:

PRINT #-1,   PRINT #-1,X,Y,B$,A$

Output to cassette #1 from computer

INPUT #-1   INPUT #1,X,Z,B$

Input from cassette #1 to computer.

If you are having trouble with keyboard bounce (double letters from a single key stroke), use the debounce tape. More recent keyboards seem not to have as much trouble with this as did those Level II's delivered before 1980.

LESSON 7
Games
and Playtoys
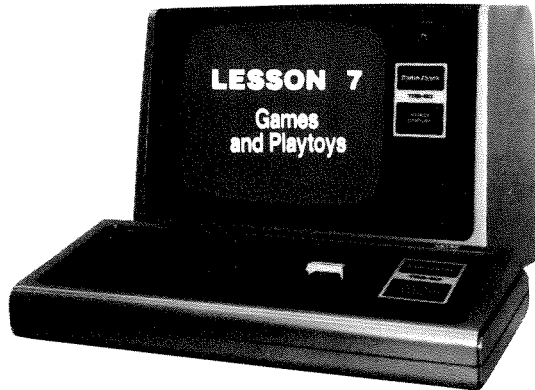
GAMES AND PLAYTOYS

Computers are magnificient game players. It isn't hard to <u>play</u> games which someone else has programmed – if they have done a good job of writing the program, but writing quality game-playing programs is difficult.

Here is a not very well written game. Put it on your computer; play it two or three times and then come back to the text and together we shall improve it.

```
 5  REM       TREASURE HUNT - FIRST TRY
1∅  CLS
2∅  X1 = RND(5∅) : Y1 = RND(5∅)
3∅  PRINT "A TREASURE IS BURIED IN A 50 FOOT BY 50 FOOT SQUARE.  IF YOU
    CAN LOCATE THE GRID POINT NEAREST IT, YOU WILL FIND THE TREASURE."
4∅  PRINT
5∅  INPUT "PLEASE TYPE COORDINATES  X, Y = ";X,Y
6∅  IF X = X1 AND Y = Y1 THEN 5∅∅
7∅  A = X - X1 : B = Y - Y1
8∅  D = SQR(A*A+B*B)
9∅  PRINT "SORRY YOU MISSED IT BY";D;"FEET."
15∅  GOTO 4∅
5∅∅  REM       TO HERE ONLY IF TREASURE IS FOUND
51∅  FOR Q = 1 TO 8∅
52∅     PRINT @ RND(1∅∅∅), "WHEEEE YOU GOT IT."
53∅  NEXT Q
54∅  GOTO 1∅
```

Well – it's a game – and the program works if you typed it in correctly. But it isn't much fun. You really need more information than just the distance to the treasure.

It would help if the program told you whether X and Y are too big or too small. Do you want to try to revise the program on your own before you continue?

One of the most common games computer programmers play is "one-up-man-ship". Modifying existing programs is one form of the sport. An important way to gain programming skill is to adapt and improve a working program written by someone else. There is an axiom among computer programmers that says

> Any program you have written,
> I can improve upon.

It has as its converse

> Any program I have written,
> you can improve upon.

and together they have a corollary

> Any program I have written,
> I can improve upon.

Each of these is probably true. In some cases the real question is whether or not it is worthwhile to improve on a program even if you can do so.

The answer depends upon how frequently the program will be run, and how much of your time will be required to improve the program.

Some ways it is frequently possible to improve programs are:

1. Make them use less computer time.
2. Provide output better suited to the user's needs.
3. Make the program easier to understand (and hence easier to modify and maintain).
4. Generalize the program so it is useful in solving other problems without sacrificing speed.

Programs designed to play games or solve puzzles somehow seem particularly subject to improvement, even by beginners. We shall look at several commercially available programs and how they can be improved. This is one of the most satisfying ways to improve your own ability as a programmer.

Let's go back and improve the treasure hunt program. The "heart" of the original program is

.

.

.

```
 5Ø   INPUT "PLEASE TYPE COORDINATES  X,Y =";X,Y
 6Ø   IF X = X1 AND Y = Y1 THEN 5ØØ
 7Ø   A = X - X1  :  B = Y - Y1
 8Ø   D = SQR(A*A+B*B)
 9Ø   PRINT "SORRY YOU MISSED IT BY";D;"FEET."
15Ø   GOTO 4Ø
5ØØ   REM  TO HERE ONLY IF TREASURE IS FOUND
```

.

.

.

```
54Ø   GOTO 1Ø
```

It would be nice to print another line of type right after 9Ø that tells us which way to go. Let's insert the following:

```
1ØØ   IF A > Ø   THEN A$="TOO BIG" ELSE A$="TOO SMALL"
11Ø   PRINT "YOUR X VALUE IS ";A$,
12Ø   IF B > Ø   THEN A$ = "TOO BIG" ELSE A$ = "TOO SMALL"
13Ø   PRINT "YOUR Y VALUE IS ";A$
```

Insert these instructions and run the program again. It is more fun this way, isn't it? Play it several times.

HEY – DID YOU NOTICE THAT THE PROGRAM CHEATS? YES IT DOES.

If you get either X or Y correct, but not both correct, it tells you to change them both.

Insert temporary debugging instruction

```
 21  PRINT  X1, Y1
```

and run it again. The correct treasure location will now be printed at the top of the screen so you can test it. Try putting in two wrong values – Seems OK. Next put in the correct value for one coordinate and the wrong one for the other coordinate. Hmmm – The program still claims both values are either TOO BIG or TOO SMALL. That won't do, will it?

OK   OK   OK

We'll fix that too. Try

```
1ØØ   IF A > Ø   THEN A$ = "TOO BIG" ELSE IF A = Ø THEN A$ = "OK" ELSE
      A$ = "TOO SMALL"
12Ø   IF B > Ø   THEN A$= "TOO BIG" ELSE IF B = Ø THEN A$ = "OK" ELSE
      A$ = "TOO SMALL"
```

115

Try it – it works now. Remember to delete instruction 21 before you show it off to someone else.

There is really no reason we need always work on a 50 by 50 foot square. How about letting the player select the size of the square? We'll need an instruction to INPUT the size of the square.

15   INPUT "PLEASE TYPE LENGTH OF SIDE OF SEARCH SQUARE.";S

Remember to thwart the person who puts in 4.73 or –2 or some such joke.

16   S = INT(ABS(S))

Before continuing you should look through the program and see what other instructions need to be modified.

Instructions 2∅ and 3∅ need to be changed; our new program (still not perfect) is

```
 5   REM       TREASURE HUNT – SECOND TRY
1∅   CLS
15   INPUT "PLEASE TYPE LENGTH OF SIDE OF SEARCH SQUARE.";S
16   S = INT(ABS(S))
2∅   X1 = RND(S) : Y1 = RND(S)
3∅   PRINT "A TREASURE IS BURIED IN A";S;" BY ";S;"FOOT SQUARE.  IF YOU
     CAN LOCATE THE GRID POINT NEAREST IT, YOU WILL FIND THE TREASURE."
4∅   PRINT
5∅   INPUT "PLEASE TYPE COORDINATES  X,Y =";X,Y
6∅   IF X = X1 AND Y = Y1 THEN 5∅∅
7∅   A = X – X1 : B = Y – Y1
8∅   D = SQR(A*A+B*B)
9∅   PRINT "SORRY YOU MISSED IT BY";D;"FEET."
1∅∅  IF A > 0 THEN A$ = "TOO BIG" ELSE IF A = ∅ THEN A$ = "OK" ELSE
     A$ = "TOO SMALL"
11∅  PRINT "YOUR X VALUE IS ";A$
12∅  IF B > ∅ THEN A$ = "TOO BIG" ELSE IF B = ∅ THEN A$ = "OK" ELSE
     A$ = "TOO SMALL"
13∅  PRINT "YOUR Y VALUE IS ";A$
15∅  GOTO 4∅
5∅∅  REM    TO HERE ONLY IF TREASURE IS FOUND
51∅  FOR Q = 1 TO 8∅
52∅     PRINT @ RND (1∅∅∅),  "WHEEE  YOU GOT IT."
53∅  NEXT Q
54∅  GOTO  1∅
```

Play the game a few times and then, after correcting the spacing on instructions 3∅, 1∅∅ and 12∅ if needed, show it to a friend.

What happens if someone inserts .5 or ∅ as the value for S? Maybe we need another smart aleck guard at 17.

17 IF ABS(S) = ∅ THEN S = 5∅∅ : PRINT "ALL RIGHT, SMARTY, I'LL FIX YOU.
    TRY THIS."

Modifications of this treasure hunt program can provide the first
approximation of a number of much more interesting games.

How about adding <u>Depth</u> at which the treasure is buried as well as
X,Y locations?

Could you modify the coordinate game to play the child's game
"BATTLESHIP" in which the computer selects positions for several targets
which you must locate?

Can you insert graphics which will show you where your choices have
been and the corresponding D value rather than telling you which way to
move?

Try a few modifications of your own.

Here are some games programmed (or adapted) by secondary school
students in Oklahoma in Spring, 1979. If you find unfamiliar instructions,
see Lesson 16 or your TRS-80 LEVEL II BASIC Reference Manual.

```
  5  CLS
  7  PRINT "THIS IS A GUESSING PROGRAM.  YOU ARE TO GUESS A FRACTION N/D
     WITH 1<=N<=10, 1<=D<=10."
  9  PRINT
1∅∅  G = RND(1∅)/RND(1∅)
11∅  PRINT "I HAVE A NEW FRACTION FOR YOU TO GUESS.  PLEASE TYPE YOUR
     GUESS IN DECIMAL FORM."
12∅  INPUT X
13∅  IF X = G THEN PRINT "WHEEE...YOU GOT IT!": GOTO 1∅∅
14∅  IF X > G THEN 17∅
15∅  PRINT "YOUR GUESS IS TOO SMALL.  TRY AGAIN PLEASE."
16∅  GOTO 12∅
17∅  PRINT "YOUR GUESS IS TOO BIG.  PLEASE TYPE IN ANOTHER GUESS."
18∅  GOTO 12∅
```

Play the game; then consider any desirable changes. One student suggest-
ed adding the following: *(However, it did not work as anticipated. Can
you help?)*

```
1∅  ON ERROR GOTO 9∅∅
9∅∅  PRINT "PLEASE TYPE THE FRACTION USING DECIMAL FORM, NOT N/D FORM."
91∅  GOTO 9
```

```
  3   REM     KAREN JEANNE HENRY
  4   REM     R1  BX 580 MCLOUD, OK 74851
  5   REM     (405) 273-1133
 10   REM     THE HANGMAN PROGRAM IS SELF EXPLAINED.  THE PERSON PLAYING
              WILL INPUT A LETTER AND THE COMPUTER WILL TELL YOU IF YOU
              ARE CORRECT OR NOT.
 20   REM     AS YOU INPUT EACH LETTER THE COMPUTER MAY OR MAY NOT CHANGE
              GRAPHICS
 25   REM     WHEN YOU FINISH THE GAME THE COMPUTER WILL VERIFY YOUR WIN OR
              LOSS BY CHANGING GRAPHICS AND/OR WORDS
 30   REM     X + Y ARE USED IN SET STATEMENTS IN FOR-NEXT LOOPS
 31   REM     D$( ) ARE THE WORDS, F$( ) ARE THE LETTERS OF THE WORDS
 32   REM     WD IS THE TOTAL NUMBER OF WORDS IN THE MAIN PROGRAM
 33   REM     W$ IS THE INPUTED LETTER L(N) + F1 ARE COUNTING DEVICES
 34   REM     K IS A VARIABLE OF HOW MANY WRONG LETTERS
 99   CLEAR 100
100   REM
102   CLS
103   WD = 10
104   K=0: FOR N=1 TO 8: L(N)=0: NEXT N
105   PRINT "**HANGMAN**"
106   FOR X = 9 TO 51 STEP 6
107   SET(X,15): SET(X + 1,15) : SET (X + 3,15) : SET(X + 2,15): NEXT X
108   FOR X = 70 TO 79: SET(X,44): NEXT X
109   FOR Y = 4 TO 44: SET(74,Y): SET(75,Y): NEXT Y
110   FOR X = 76 TO 101: SET(X,4): NEXT X
140   FOR Y = 5 TO 7: SET(101,Y): NEXT Y
150   IF FG = 1 THEN 160 ELSE D$(1) = "APPLE": D$(2) = "HEART": D$(3) =
              "STUDENT" : D$(4) = "TEACHER": D$(5) = "PENCIL" :D$(6) = "PAPER" :
              D$(7) = "PEN" : D$(8) = "WORK": D$(9) = "TIRED": D$(10) = "HARD"
160   DD = DD + 1: IF DD> WD THEN 2050 ELSE Z$ = D$(DD)
180   N = LEN(Z$)
190   F$(1) = MID$(Z$,1,1)
200   F$(2) = MID$(Z$,2,1)
210   F$(3) = MID$(Z$,3,1)
220   F$(4) = MID$(Z$,4,1)
230   F$(5) = MID$(Z$,5,1)
240   F$(6) = MID$(Z$,6,1)
250   F$(7) = MID$(Z$,7,1)
260   F$(8) = MID$(Z$,8,1)
270   IF F$(3)<>"" THEN 290
280   RESET (21,15) : RESET (22,15) : RESET (23,15) : RESET (24,15)
290   IF F$(4) <>"" THEN 310
300   RESET (27,15) : RESET (28,15) : RESET (29,15) : RESET (30,15)
310   IF F$(5)<>"" THEN 330
```

```
320  RESET (33,15) : RESET (34,15) : RESET (35,15) : RESET (36,15)
330  IF F$ (6) <> " " THEN  350
340  RESET (39,15) : RESET (40,15) : RESET (41,15) : RESET (42,15)
350  IF F$ (7) <> " " THEN 370
360  RESET (45,15) : RESET (46,15) : RESET (47,15) : RESET (48,15)
370  IF F$ (8) <> " " THEN 390
380  RESET (51,15) : RESET (52,15) : RESET (53,15) : RESET (54,15)
390  PRINT @ 388, "THIS WORD HAS "; N; "LETTERS",;
400  PRINT @ 452, "TYPE EACH LETTER" ;
410  PRINT @ 516, "PRESSING ENTER EACH TIME" ;
415  Z = 644
420  PRINT @ 896, " " ;
425  INPUT W$
427  PRINT @ 980, " " ;
430  FOR N = 1 TO 8
440  IF W$ = F$(N) THEN PRINT @ (258 +(3*N)), W$; : L(N) = 1 : FLAG = 1 :
     GOTO 450
450  NEXT N
460  IF FL = 0 THEN PRINT @ Z, W$; : Z = Z + 2
470  P = 0
480  FOR N = 1 TO 8 : IF L(N) = 1 THEN P = P + 1
500  NEXT N
510  IF P = LEN(Z$) THEN 2000
520  IF FL = 1 THEN FL = 0 : GOTO 420
530  ON K + 1 GOTO 540, 660, 710, 810, 910, 980
540  K = 1
550  FOR X = 95 TO 107 : SET (X,8): SET (X,16) : NEXT X
580  FOR Y = 10 TO 14 : SET (109,Y) : SET (93,Y) : NEXT Y
610  SET (94,9) : SET (108,9) : SET (108,15) : SET (94,15)
620  SET (97,10) : SET (105,10) : SET (101,11) : SET (101,12)
630  SET (95,12) : SET (96,13) : SET (97,14) : SET (98,14) : SET (99,14) :
     SET (100,14)
640  SET (101,14) : SET (102,14) : SET (103,14) : SET (104,14) :
     SET (105,14) : SET (106,13) : SET (107,12)
650  GOTO 420
660  K = 2
670  FOR Y = 17 TO 33 : SET (100,Y) : SET (101,Y) : SET (102,Y) : NEXT Y
700  GOTO 420
710  K = 3
720  FOR X = 103 TO 112 : SET (X,20) : NEXT X
750  Y = 19 : SET (117,14)
760  FOR X = 112 TO 116 : SET (X,Y) : SET (X+1,Y) : Y = Y-1 : NEXT X
800  GOTO 420
810  K = 4
820  FOR X = 90 TO 99 : SET (X,20) : NEXT X
850  Y = 26
860  FOR X = 85 TO 90 : SET (X,Y) : SET (X,Y-1) : Y = Y-1 : NEXT X
900  GOTO 420
```

```
910   K = 5 : Y = 33
920   FOR X = 103 TO 112 : SET (X,Y) : SET (X+1,Y) : Y = Y+1  : NEXT X
960   SET (113,43) : SET (114,43) : SET (115,43) : SET (116,43) :
      SET (117,43)
970   GOTO 420
980   Y = 42
990   FOR X = 89 TO 98 : SET (X,Y) : SET (X+1,Y) : Y = Y-1 : NEXT X
1030  SET (85,43) : SET (86,43) : SET (87,43) : SET (88,43) : SET (89,43)
1040  RESET (101,12) : RESET (95,12) : RESET (96,13) : RESET (106,13) :
      RESET (107,12)
1060  FOR X = 97 TO 104 : RESET (X,14) : NEXT  X
1100  SET (97,14) : SET (98,13) : SET (99,13) : SET (100,13) :
      SET (101,13) : SET (102,13) : SET (103,13) : SET (104,13) :
      SET (105,14)
1110  PRINT @ 772, " AHHH! THAT'S TOO BAD"; : GOTO 2010
2000  PRINT @ 772, "CONGRATULATIONS! YOU WON!!!";
2010  FOR N = 1 TO 2000  : NEXT N : CLS : GOTO 100
2050  REM      THIS CHANGES WORDS
2055  ON  F+1    GOTO 2060, 2070
2060  D$(1) = "GEOMETRY" : D$(2)="ALGEBRA" : D$(3)= "TRIANGLE" :
      D$(4)= "FORMULA" : D$(5)= "SOLUTION" : D$(6)= "SUBTRACT" :
      D$(7)= "NEGATIVE" : D$(8)= "REDUCE" : D$(9)= "INVERSE" :
      D$(10)= "FRACTION" : K=1 :  F=1 :  GOTO 170
2065  DD = 0 : FG = 1 : GOTO 100
2070  REM      THE PERSON USING THIS PROGRAM CAN ADD MORE WORDS IN THIS SAME
      WAY, USING THE D$(SUBS), OR HE CAN USE DATA FILES AND STORE WORDS ON
      A TAPE RECORDER CONTROLLED BY THE COMPUTER.  USE   FOR N = 1 TO 10 :
      INPUT # -1 , D$(N) : NEXT N : GOTO 2065
```

This program <u>will</u> fit on a 4-K Level II TRS-80, but you may need to "squeeze out" <u>most</u> of the spaces as suggested in Lesson 11.

```
1060 FORX=97TO104:RESET(X,14):NEXTX
```

is hard to read, but requires less computer memory. Lines 3 to 34 may be ommitted, if necessary.

After it is up and working, you may wish to try to improve it by including additional words from a cassette file. See Lesson 6.

```
 10   REM      * REVERSE *        BOB YARBROUGH   4-78
 20   REM      RANDOM DIGITS ARE FLASHED.  OPERATOR MUST ENTER NUMBER
 21   REM      FORMED BY WRITING DIGITS IN REVERSE ORDER
 30   REM      L IS INITIAL LENGTH OF STRING OF DIGITS
 31   L=3
 40   REM      A IS LENGTH OF COUNTER
 41   A=1000
 50   REM      G IS NUMBER OF CORRECT RESPONSES BEFORE STRING
 51   REM      IS LENGTHENED
 52   G=3
 60   REM      STRING IS SHORTENED WHENEVER AN ERROR IS MADE.
 70   CLS
 80   PRINT "YOU MUST ENTER DIGITS SHOWN IN REVERSE ORDER."
 90   PRINT
100   C=0
101   PRINT "NEXT STRING WILL BE ";L;" LONG."
102   Z=0
104   GOSUB 200
106   CLS
110   FOR X=57 TO 68
112       SET(X,21)
114       SET(X,29)
116   NEXT X
120   FOR Y=21 TO 29
122       SET(57,Y)
124       SET(68,Y)
126   NEXT Y
130   FOR N=1 TO L
132       A(N)=RND(9)
134       Z=10*Z + A(N)
136   NEXT N
140   FOR N=L TO 1 STEP -1
142       PRINT @ 542,A(N);
144       GOSUB 200
145       SET(60,25)
146       SET(61,25)
147   NEXT N
148   CLS
149   PRINT
150   INPUT P
152   IF P <> Z THEN 180
160   PRINT "CORRECT"
162   C=C+1
164   IF C < G THEN 102
166   L=L+1
168   GOTO 100
```

121

```
18Ø   PRINT "WRONG    ***";Z;"***"
183   GOSUB 2ØØ
186   L=L-1
19Ø   GOTO 1ØØ
2ØØ   FOR M=1 TO A
2Ø2   NEXT M
2Ø4   RETURN
21Ø   END
```

Note: If you wish the digits to be displayed for a shorter period of
       time, change line 41 to:

 41   A = 5ØØ or some smaller value.

```
1    REM    *ASTEROIDS*    MICHAEL BRIGGS        1-79
2    CLS : PRINT " MANEUVER THE SHIP THROUGH THE ASTEROIDS BY USING THE
     UP-ARROW KEY TO GO UP, THE DOWN ARROW TO GO DOWN, AND THE < AND >
     KEYS FOR LEFT AND RIGHT."
3    Z = 15
4    DIMA(Z,15)
5    PRINT
6    INPUT "PRESS ENTER TO START" ; D
7    LL = 15Ø
9    CLS
1Ø   X = 63 : Y = 44
15   TI = Ø
2Ø   SET (X,Y) : SET (X,Y+1) : SET (X-1,Y+2) : SET (X+1,Y+2)
22   FOR G = Ø TO 127
24      SET (G,47)
26   NEXT
3Ø   FOR L = 3 TO 15
4Ø      FOR P = 1 TO Z
5Ø         A(P,L) = RND(128)-1
6Ø         SET(A(P,L),2*L)
7Ø   NEXT : NEXT
8Ø   PRINT @ 32,    "GO!"
9Ø   FOR L = 3 TO 13
1ØØ     R = RND(3)+2
11Ø     FOR P = 1 TO Z
12Ø  S$ = INKEY$
122     TI = TI + 1
123     IF S$ = "" THEN 24Ø
125     RESET (X,Y) : RESET (X,Y+1) : RESET (X-1,Y+2) : RESET (X+1,Y+2)
13Ø     IF S$ = "↑" THEN Y = Y-1 : GOTO 17Ø
14Ø     IF S$ = "↓" THEN Y = Y+1 : GOTO 17Ø
15Ø     IF S$ = "." THEN X = X+2 : GOTO 17Ø
16Ø     IF S$ = "," THEN X = X-2
17Ø     IF X = 127 OR X = 1 GOTO 5ØØ
18Ø     IF Y = 45 THEN GOSUB 1ØØØ
19Ø     IF Y = 1 THEN 2ØØØ
2ØØ     IF POINT(X,Y) OR POINT(X,Y+1) OR POINT(X-1,Y+2) OR
        POINT(X+1,Y+2) THEN 15Ø5
235     SET (X,Y) : SET (X,Y+1) : SET (X-1,Y+2)  : SET (X+1,Y+2)
24Ø     RESET (A(P,L), 2*L)
25Ø     A(P,L) = A(P,L)+R
26Ø     IF A(P,L) >=128 THEN A(P,L) = A(P,L) -128
265     IF ABS(A(P,L)-X) < 2 THEN 9ØØ
268     SET(A(P,L),2 * L)
27Ø     IF TI = LL THEN 25ØØ
28Ø  NEXT : NEXT
3ØØ  GOTO 9Ø
```

Note: NEXT works as well as NEXT X, but the practice of omitting the variable can cause a problem in nested FOR...NEXT loops.

Note: The ↓ won't show on the screen, but it is in there.

```
500    CLS
510    PRINT  "YOU HAVE JUST RUN INTO A BLACK HOLE, SORRY!"
520    GOTO 2020
900    IF 2*L    >= Y AND 2*L   <= Y+3 THEN 1505

910    GOTO 268
1000   CLS
1010   PRINT  " YOU HAVE CRASH-LANDED, BUT IT'S NOT TOO FAR TO THE BASE."
1020   GOTO 2020
1505   FOR BL= 1 TO 40 : RESET (X,Y) : RESET (X,Y+1) : RESET (X-1,Y+2) :
       RESET (X+1,Y+2) : SET (X,Y) : SET (X,Y+1) : SET (X-1,Y+2) :
       SET (X+1,Y+2) : NEXT : CLS
1510   PRINT  "(AP) HOUSTON - NASA MISSION CONTROL REPORTS THE LOSS OF AN
       INTER-PLANETARY SHUTTLE IN ROUTE TO SATURN.  THE VESSEL APPARENTLY
       SUFFERED SERIOUS ASTEROID DAMAGE AND WAS UNABLE TO RETURN TO EARTH."
1520   GOTO 2020
2000   CLS
2010   PRINT  "EXCELLENT NAVIGATION! CONGRATULATIONS ARE IN ORDER."
2015   PRINT  "YOU STILL HAVE" ; (LL-TI)*10  ; "GALLONS OF FUEL LEFT."
2020   PRINT : INPUT  "PRESS ENTER TO TRY AGAIN" ; D
2030   GOTO 9
2500   CLS : PRINT "YOU HAVE RUN OUT OF FUEL.  YOU WILL CRASH-LAND IN
       APPROXIMATELY 15 MINUTES.  WE MIGHT EVEN SEND A SEARCH PARTY TO
       PICK UP THE PIECES." : GOTO 2020
```

Here is a brief program that clutters up the screen in an interesting
fashion. Try it. You might like it.

```
100 POKE 16396,23
110 CLS
120 FOR N=14336 TO 15360
130    PRINT CHR$(PEEK(N)) ; " ";      Don't overlook the final ; here.
140 NEXT N
150 GOTO 100
```

Then type RUN ˙[ENTER] and hit several keys at once without using [ENTER] .
Try [Q] [X] [W] and then try various other keys with some or all of the
above.

←	S
7	S
W	C

To terminate the program, open the little door on the back left-hand side
of your keyboard unit and depress the button on the extreme left side.
You may need to turn your computer off and then back on again to assure
proper functioning of all keys.

You will find dozens of game programs in Softside, Computronics, Creative Computing, TRS-80 Computing (San Luis Rey, CA), PROG-80 (Milford, NH) and many other journals mentioned in Lesson 15. Some commercial games are rather expensive, costing $25 and more. Such games are still cheaper than ones you design, write and debug, if your time is valuable. Other tapes are available at a cost so low that you can hardly afford the time to type in the program and correct your typing errors. People's Software will send you a list of available pro- gram tapes (1981 price $10.95 plus postage in U.S. for some tapes). They add more to the list all the time and will swap you free tapes for good original programs which you have written.

Tape 1 ($11) contains a whole collection of games, some trivial, some excellent. Other programs deal with business, banking, bio-rhythms, speed-reading, math refresher, etc. Tape 2 ($11) contains some slightly more sophisticated banking and business programs, math programs, statist- ical programs, etc. Tape 5 ($11) is more of the same. Tape 6 is People's Pascal, Version II, and the cost is $23.50. You may or may not be ready for the Pascal language.

You can learn a lot by ordering one of these tapes and then examining, adapting and improving the programs to suit your whim. BE SURE TO SPECI- FY WHETHER YOU HAVE LEVEL I OR LEVEL II BASIC. The address is:

People's Software
Computer Information Exchange
Box 158
San Luis Rey, CA 92066
Telephone: (714) 757-4849 (VISA or MASTERCHARGE accepted for phone
                              orders)

1. Improve the treasure hunt program introduced in this lesson.

2. Either devise a number guessing game of your own or improve the game written by a secondary school student in this lesson.

3. Play Karen Henry's modified Hangman program and then modify it to use your cassette tape recorder to provide additional words.

4. Either write a game-playing program of your own or improve a game-playing program written by someone else. Almost any game-playing program can be modified and improved. Try out your own ideas.

5. See if you can determine where the "fuel supply" is stored in the ASTEROIDS program. Change the fuel allotment. Also, determine whether fuel is used up by "time elapsed" or by "number of keystrokes", and plan your playing strategy to take advantage of this knowledge.

6. Modify Bob's program to play REVERSE so the player can INPUT a value to determine the amount of time the numbers are displayed (the difficulty level).

7. Look in one of the journals listed in Lesson 15 and find a computer game. Put the game on your computer, debug and play it. Then, see if you can improve it.

LESSON 8
Edit Instructions

EDIT INSTRUCTIONS

The simplest way to change short lines in a program is certainly just to retype it, as we have been doing.

If you mistyped a program as

```
1000    FOR K = 1 BO  17      (error)
1010      PPRINT K;           (error)
1020    NEXT K
```

RUN | ENTER |

The computer will display

```
?  SN ERROR IN 1000
READY
1000_
```

By depressing key ⎣L⎦ the computer will list line 1000 and again display the line number, to facilitate correction.

```
?SN ERROR IN 1000
READY
1000 FOR K = 1 BO 17
1000_
```

You may now either space over to just under the error using the space bar, or you may depress ⎣S⎦ and the ⎣B⎦ (the character in error). In either case, you will now have

```
?SN ERROR IN 1000
READY
1000 FOR K =1 BO 17
1000 FOR K =1 _
```

Next, depress C for "change" and depress T followed by ENTER to correct the line.

Type LIST ENTER to see that the change was made.

1000 FOR K = 1 TO 17
1010   PPRINT K;
1020 NEXT K

If you type RUN ENTER you can go through a similar error detection and error correction routine for line 1010, but if you notice the "PP" when you LIST it, it is easier just to retype it as

1010 PRINT K; ENTER

If you didn't note the error and type RUN ENTER , you will obtain

```
RUN
?SN ERROR IN 1010
READY
1010 _
```

Upon depressing L the last line will show:

1010 PPRINT K;
1010 _

If you space the underline (cursor) over to directly under either of the letter P's

1010 PPRINT K;
1010 P_

and depress key D , the computer will display:

```
1010  PPRINT K;
1010  P!P!
```

The !! indicates the portion that will be deleted when you depress ENTER .

Depress ENTER

Type LIST ENTER and, sure enough, it shows:

```
1000    FOR K = 1 TO 17
1010        PRINT K;
1020    NEXT K
```

and RUN [ENTER] produces the expected result.

```
1 2  3  4  5  6  7  8  9  10  11  12  13  14  15  16  17
READY
> _
```

You can always go into the EDIT mode to change a line by typing EDIT *line number.* For example, to change the 17 in line 1000 to a 37, one merely types:

EDIT 1000 [ENTER]

The computer responds

1000 _

Depress [L] , and you will have

```
>EDIT 1000
1000 FOR K = 1 TO 17
1000 _
```

Note that the "1" of 17 is the second "1" in line 1000, so depress

[2] and then [S] and then [1]   (*Literally,* "*Seek the 2nd
                                          character* 1".)

The computer displays

```
>EDIT  1000
1000 FOR K = 1 TO 17
1000 FOR K = 1 TO _
```

Now, depress [C] followed by [3] which gives

1000 FOR K = 1 TO 3_

Depressing [ENTER] will complete the line and enter the changes into the program, as   LIST [ENTER] will then show, and RUN [ENTER] will demonstrate.

Chapter 9 in your TRS-80 Level II BASIC Reference Manual gives additional details, which are summarized here.

EDIT MODE SUBCOMMANDS AND FUNCTION KEYS

Subcommand/Function Key	Function
EDIT *line no.* ENTER	where *line no.* is a valid line number containing an instruction. This sends the computer to EDIT that line.
ENTER	End editing and return to Command Mode.
SHIFT ↑	Escape from subcommand and remain in Edit Mode.
n Space-Bar	Move cursor *n* spaces to right.
n ←	Move cursor *n* spaces to left.
L	List remainder of program line and return to beginning of line.
X	List remainder of program line, move cursor to end of line, and start Insert subcommand.
I	Insert the following sequence of characters at current cursor position; use SHIFT ↑ to get out of insert subcommand.
A	Cancel changes and return cursor to beginning of line.
E	End editing, save all changes and return to Command Mode. Same as ENTER
Q	End editing, cancel all changes made and return to Command Mode.
H	Delete remainder of line and insert following sequence of characters; use SHIFT ↑ to get out of this insert subcommand.
*n*D	Delete specified number of characters *n* beginning at current cursor position.
*n*C	Change (or replace) the specified number of characters *n* using the next *n* characters entered.
*n*Sc	Move cursor to *n*th occurrence of character *c*, counting from current cursor position.

n $\boxed{K}$ c Delete all characters from current cursor
position up to nth occurrence of character c ,
counting from current cursor position.

Let us try to use some of these EDIT instructions. Type along on
your TRS-80 to help you follow what is taking place.

If you have the program line:

1ØØ PLEASE PRINT THIS LINE.

and type

RUN $\boxed{\text{ENTER}}$

your screen will show

```
>RUN
?SN ERROR IN 1ØØ
READY
1ØØ _
```

If you depress $\boxed{L}$ the screen will show

```
>RUN
?SN ERROR IN 1ØØ
READY
1ØØ  PLEASE PRINT THIS LINE.
1ØØ _
```

By now you realize that you need a PRINT instruction and that the message
needs to be in quotes if it is to be printed.

Depress $\boxed{I}$ key   (Nothing seems to happen, but it has put the
                                            editor into the insert mode.)

Type
PRINT "   (which will appear on the screen)

Depress $\boxed{\text{SHIFT}}$ $\boxed{\uparrow}$ (Again nothing seems to happen, but this
                                            releases you from the insert mode.)

Depress $\boxed{X}$ (This moves to the end of the line and puts
                                          the editor back into insert mode.)

Your TRS-80 will now show

```
?SN ERROR IN 100
READY
100 PLEASE PRINT THIS LINE
100 PRINT "PLEASE PRINT THIS LINE._
```

Note that the underscore at the end of the line shows you where the next
character will appear. It is right where you need to insert another quote.
To do so, type

" [ENTER]

Your correction is now complete.

If you type

RUN [ENTER]

the program will print the desired message.

Note: If, back at the beginning when you ran 100 PLEASE PRINT THIS LINE
and your screen showed

```
>RUN
?SN ERROR IN 100
READY
100 _
```

you forgot to give the EDITOR a command by depressing [I] (or some other
command key) and just started to type PRINT, an unexpected action would
have taken place.

The editor would ignore [P] [R] as unacceptable commands. It would then
use [I] as the "insert" command and would have inserted N T.

This is not at all what you wanted.

EDIT takes a bit of care, but it is well worth the effort.

LESSON 9
Simulation

SIMULATION

A SIMULATION is a charade that uses a mathematical model to repre-
sent life-like experiments. The quality of the simulation is highly
dependent upon how well the mathematical model represents the actual phys-
ical or social situation under study.

Many times it is difficult to compute a result of an actual experi-
ment, either because the computation itself is too difficult to perform,
or because the theory is not yet sufficiently developed. Even if the
theory isn't well developed, it may still be feasible to use simulation
to study the situation.

Simulation is one of the most important uses of modern computers. A
simulation is a caricature. As in any caricature, it emphasizes the most
prominent features under consideration and ignores the others.

We shall not attempt to simulate complex processes here. Instead,
we shall simulate several rather simple situations -- problems which you
quite possibly could solve without resorting to simulation, but which will,
nonetheless, illustrate the spirit of the game.

The heart of most simulation processes is some sort of random number
generator.

The TRS-80 has an easy-to-use random number generator which you al-
ready investigated and used in Lesson 4.

RND($\emptyset$) produces a random decimal between 0 and 1, not including
  either endpoint.
RND(K) produces a random integer between 1 and K, including both
  endpoints.
RANDOM is sometimes used once at the very beginning of programs that
  will be used frequently. It reseeds the random number gen-
  erator so that the same sequence of RND ( ) values is not
  produced on each run.

133

Let's begin with a simple program to simulate five rolls of two dice. Each die will produce one of the values 1, 2, 3, 4, 5, 6 at random. We wish to display the value shown on each die as well as their sum. For convenience, the individual dice are represented by R and G (for Red and Green, to tell them apart).

```
100   REM   DICE THROWING PROGRAM
110   FOR K = 1 TO 5
120      R = RND(6)
130      G = RND(6)
140      S = R + G
150      PRINT  "DICE SHOW" ; R ; G , "SUM =" ; S
190   NEXT K
```

Get this up and running before you go further.

Now imbed the above in a program that will collect statistics on how frequently each sum occurs in 100 rolls.

```
10   DIM T(12)
20   REM        ZEROS T(2) TO T(12) FOR FUTURE RECORDS OF TOTALS
30   FOR S = 2 TO 12
40      T(S) = 0
50   NEXT S

100   REM   DICE THROWING PROGRAM
110   FOR K = 1 TO 100
120      R = RND(6)
130      G = RND(6)
140      S = R + G
150      PRINT  "DICE SHOW" ; R ; G , "SUM =" ; S
160      T(S) = T(S) + 1  :   REM  THIS ACCUMULATES TOTALS IN T( )
190   NEXT K

300   REM        PRINTING OF TOTALS
310   PRINT
320   FOR S = 2 TO 12
330      PRINT S ; "WAS ROLLED" ; T(S) ; "TIMES IN 100 ROLLS OF TWO DICE."
340   NEXT S
350   PRINT
```

Once the program is running properly, you may change instruction 110 to
```
110   FOR K = 1 TO 1000
```
to obtain a larger sample size. You will wish to change 100 to 1000 in instruction 330 as well. Additional speed may be obtained by deleting instruction 150 once you are sure the program is functioning as desired.

134

A BIOLOGICAL SIMULATION

Example 9-1

Suppose a biologist takes 100, equal-sized drops of liquid from a
liter flask containing bacteria which are randomly distributed (i.e.,
no tendency to cluster). Upon testing, it is discovered that 50 of the
samples are bacteria-free, while the other 50 contain 1 or more bacteria
per sample.* Assuming that the bacteria are distributed at random among
the samples, approximately what is the total number of bacteria in the
100-drop sample?

The exact answer <u>cannot</u> be determined. However, it is possible to
obtain a statistical estimate of the desired number. You may or may not
have sufficient mathematical sophistication to solve this problem
directly, but it is an easy matter to simulate it.

Number 100 locations V(0) to V(99), and store zero in each. See
Lesson 10 for additional information on subscripted variables. Then
generate a sequence of random two-digit numbers. Each time the number N
is generated ($0 \le N \le 99$) we add one to location V(N). This is con-
tinued until exactly 50 of the locations contain non-zero numbers and 50
still contain zeros. At this stage, we have an estimate of the total
number of bacteria in the 100 samples, either by having kept track, or
by summing the V(N)'s.

Of course, the estimate may not be very good if we perform the
experiment only once. By running it several times, using different ran-
dom number sequences, we can obtain an average estimate that may be
reasonably close. The "proper" number of runs is a mathematical problem
of greater difficulty than we shall discuss here.

Do you see that this technique might be helpful? Actually, the
mathematics needed to justify and analyze techniques of this type is
highly sophisticated, but fortunately for many <u>users</u>, the techniques
themselves are comparatively simple to use. A flow-chart of a computer
program to simulate this experiment is given on the following page.

---

* The testing consists of adding a drop of reagent to the sample drop.
If it is bacteria-free, it will remain clear, but if the sample contains
one or more bacteria, it turns purple. However, the sample does not turn
twice as purple if two or more bacteria are present, so our test gives
<u>no idea of the total number of bacteria present</u>.

```
                              ┌─────────────────────┐
    ╭───────╮                 │     Initialize      │        ⎧DIM V(99) sets aside 100
    │ Start │────────────────▶└─────────────────────┘        ⎪locations V(0), V(1),...,
    ╰───────╯                 ┌─────────────────────┐        ⎨V(99) to represent the
                              │    Set starting     │        ⎪100 samples.
                              │      value of       │        ⎩
                              │    each sample      │
                              │     V(1) = 0        │
                              └─────────────────────┘
                                   ┌─────────┐                ⎧C is used to keep track
                                   │  Set    │                ⎪of how many cells now
                                   │  C = 0  │                ⎪contain one or more
                                   │  and    │────────────────⎨bacteria.
                                   │  T = 0  │                ⎪T = total number of
                                   └─────────┘                ⎩bacteria to date.

                              ┌─────────────────────┐
                              │      Generate       │
                              │  a random two-digit │
                              │      integer        │
                              │                     │
                              │     0 ≤ N ≤ 99      │
                              └─────────────────────┘

                                  ╱ Does ╲
                                 ╱ the Nth ╲
            ╭────╮             ╱ sample contain ╲        ╭─────╮
            │ No │◀───────────◀    bacteria    ▶────────▶│ Yes │
            ╰────╯               ╲   now?    ╱           ╰─────╯
              │                    ╲       ╱                 │
    ┌──────────────────┐             ╲   ╱                   │
    │   Increment      │                                     │
    │      C           │                                     │
    │   by one.        │─────────────────┐      ┌────────────┘
    └──────────────────┘                 ▼      ▼
                              ┌─────────────────────┐
                              │   Add one bacteria  │
                              │  to the Nth sample  │
                              └─────────────────────┘
                              ┌─────────────────────┐
                              │   Increment  T      │
                              │     by one          │
                              └─────────────────────┘

                                   ╱  Do   ╲
                                  ╱50 samples╲
            ╭────╮            ╱contain bacteria?╲      ╭─────╮
            │ No │◀──────────◀────────i.e.────────────▶│ Yes │
            ╰────╯               ╲  C = 50  ╱           ╰─────╯
                                   ╲   ?  ╱                 │
                                                ┌───────────────────────┐
                                                │  Summarize results    │
                                                │  of this test and     │
                                                │  print it.            │
                                                └───────────────────────┘
                                                ╭───────────────────────╮
                                                │     Possibly          │
                                                │ go back to START      │
                                                │ and RUN it again      │
                                                ╰───────────────────────╯
```

136

```
1ØØ   DIM V(99)
11Ø   FOR K = Ø TO 99
12Ø      V(K) = Ø
13Ø   NEXT K
15Ø   C = Ø
16Ø   T = Ø
```

C=Number of Cells containing bacteria.
T=Total number of bacteria.

```
2ØØ   REM   GENERATE N: Ø <= N <= 99; INCREMENT V(N),
            TOTAL, AND IF APPROPRIATE COUNT.
21Ø   N = RND(1ØØ)-1
22Ø   IF V(N)  >  Ø  THEN 23Ø
225   C = C+1
23Ø   V(N) = V(N) + 1
24Ø   T=T+1
25Ø   IF C    <  5Ø   THEN 21Ø
```

```
3ØØ   PRINT "THE 1ØØ DROPS CONTAIN A TOTAL OF" ;    T    ; "BACTERIA."
```
Before expanding this experiment, run it a few times. If it shows the
same result each time, you may need to add the instruction
```
5   RANDOM              or        5 RANDOMIZE
```

to prevent the program from resetting the random number generator to the
same place at the start of each run. After the program is debugged and
running, you could imbed it in a FOR....NEXT loop to run it, say, 100
times and give information on the 100 trials. You might even run it 1000
or 5000 times and compute mean and standard deviation of the total number
of bacteria to be expected in the 100–drop sample. You might also con-
sider how to change the program if the number of contaminated drops found
in the 100–drop sample were 35 or 70 instead of 50.

     For now merely add:
```
 9Ø   G = Ø : REM   G  WILL BE USED TO ACCUMULATE THE GRAND TOTAL OR
                       CUMULATIVE TOTAL OF BACTERIA IN 1ØØ RUNS
```

```
1Ø5   FOR S = 1 TO 1ØØ
3ØØ      PRINT "SAMPLE";S;"CONTAINS A TOTAL OF";   T   ;"BACTERIA."
31Ø      G = G + T
32Ø   NEXT S
4ØØ   PRINT
41Ø   PRINT  " THE HUNDRED RUNS PRODUCED AN AVERAGE OF "; G/1ØØ ; " TOTAL
               BACTERIA PER RUN "
```

     In Practice Set 9 you will modify this program so that the number
of runs can be specified by the user. You may wish to think a bit about
which instruction will need to be changed. Actually, the simulated
average of 100 runs should be expected to agree quite closely with the
answer of 69 bacteria obtained using statistical theory.
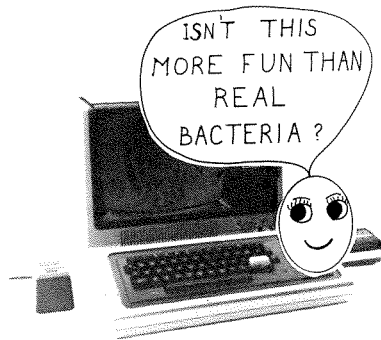
A SECOND BIOLOGICAL SIMULATION

EXAMPLE 9-2

Consider a superficially related, but actually different example. This time our biologist has a liquid which he knows contains exactly 200 amoebas (or fish) which again are distributed at random throughout the liquid. If he separates the liquid into 50 equal size samples V(1), V(2), ...,V(50), how many samples will be amoeba-free? (Also, how many will contain 1, 2, 3, 4, 5, 6, more than 6 amoebas?)

The computation on this is well within your understanding, but that is not really the purpose of this example. Let's simulate the problem by creating 50 storage cells, each of which initially contains zero amoebas. This time generate exactly 200 pseudo-random numbers N, with $1 \leq N \leq 50$. Each time the number N appears, add 1 to the number in location V(N).

```
100   DIM V(50), M(8)                Sets aside space for samples V( )
110   FOR K = 1 TO 50                and final data collection M( ).
120      V(K) = 0                    This zeros 50 samples V(K).
130   NEXT K

200   FOR B = 1 TO 200               This distributes the 200 amoebas
210      N = RND(50)                 into the 50 samples.
220      V(N) = V(N) + 1
230   NEXT B

300   REM   ZERO THE M'S TO COLLECT DATA FOR FINAL REPORT
310   FOR L = 0 TO 8
320      M(L) = 0
330   NEXT L
```

(Continued on next page)

```
4ØØ  REM   COLLECT DATA ON HOW MANY V-SAMPLES CONTAIN L AMOEBAS
41Ø  FOR B = 1 TO 5Ø
42Ø     N = V(B)
425     IF N < 7  THEN 43Ø
428     N = 8
43Ø     M(N) = M(N) + 1
44Ø  NEXT B

5ØØ  REM     PRINT OUT SUMMARY
51Ø  FOR L = Ø TO 6
52Ø     PRINT " THERE ARE " ; M(L) ; " SAMPLES WITH EXACTLY " ; L ;
        " AMOEBAS "
53Ø  NEXT L
54Ø  PRINT " THERE ARE " ; M(8) ; " SAMPLES CONTAINING 7 OR MORE AMOEBAS "
```

Once you have the program up and debugged and feel it is running well, you may wish to include it in a FOR ... NEXT loop to run the experiment several times and to tabulate the data in a more useful form. This is left for you to do. Once again it will be discovered that the average of, say 100, runs of such a simulation will agree quite closely with the forecast obtained using statistical theory, in which the number of amoeba-free samples is

$$50(1- \frac{1}{50})^{200} = 50(1+ \frac{-4}{200})^{200} \approx 50e^{-4} = 50*.0183 = .9$$

The results of our simulation program correspond very closely to the theoretical results.

A simulation of this type can also produce convincing evidence that certain phenomena are not random. Let us assume that in your area P persons died during D days of this year. If we assume that deaths occur at random, then we are placing P markers into D cells just as we were in the previous example, except P = 200, and D = 50. In this case, the agreement between the outcome of the simulation and actual data is not good. Indeed, further analysis shows that deaths are not usually distributed at random, but rather that a high "successive-day-dependence" seems to exist with alternate high and low death periods. This too can be simulated, but the model is more sophisticated.

EXAMPLE 9-3

Now examine another simulation. This is essentially an Ehrenfest model with three molecules (see your physics or chemistry teacher). For simplicity it is posed as a "Guppy" problem.

Assume your aquarium contains 3 pregnant guppies. Because of their gravid condition you prefer not to net the guppies, but instead to place a "transfer cage" in the aquarium. At the start there are no guppies in the transfer cage and three in the aquarium. As time goes on the guppies swim into or out of the transfer cage at random. When all three are in

the transfer cage, the door is closed and no further exchange is possible. The guppies are then transferred to a maternity aquarium. The number of guppies in the transfer cage at various times might be

$$0 \to 1 \to 2 \to 1 \to 2 \to 1 \to 0 \to 1 \to 2 \to 1 \to 2 \to 3 \quad \text{end.}$$

If you were unusually lucky, it might even be

$$0 \to 1 \to 2 \to 3 \quad \text{end.}$$

The problem is to determine the average number of <u>arrows</u> in the chain before all three fish are in the transfer cage.

Again there exist statistical techniques (for example Markov Chain techniques) which could be used to determine the desired result, but we prefer to simulate the process.

Before examining the solution proposed by your author, at least try your hand at flow charting the heart of this problem.

Here is <u>one</u> possible model. Use a random number generator to determine whether the next guppy goes into or out of the cage: If $RND(\emptyset) \leq .5$ put a fish into the transfer cage. If $RND(\emptyset) > .5$ move out one, if possible. When all three fish are in the cage, stop the simulation.

```
1∅   REM      FIRST GUPPY MODEL SIMULATION
2∅   C = ∅ : REM     C = # OF GUPPIES IN CAGE
3∅   A = ∅ : REM     A = # OF ARROWS
9∅   PRINT "HISTORY=";
1∅∅  R = RND (∅)
11∅  IF R >.5 THEN 2∅∅
12∅  REM      HERE IF FISH ENTERS CAGE  R< = .5
13∅  A = A + 1
14∅  C = C + 1
15∅  PRINT C;
16∅  IF C = 3 THEN 5∅∅
18∅  GOTO 1∅∅
2∅∅  REM      HERE IF FISH DOES NOT ENTER CAGE R >  .5
21∅  IF C = ∅   THEN 1∅∅
22∅  A = A + 1
23∅  C = C - 1
24∅  PRINT C;
25∅  GOTO 1∅∅
5∅∅  REM      HERE IF ALL 3 GUPPIES ARE IN CAGE
51∅  PRINT "SUCCESS AT LAST"
52∅  PRINT A;  "ARROWS OR 'SWIM THROUGHS' TOOK PLACE"
53∅  PRINT
```

After debugging the above model you may enclose it in a program to run 25 times and give the <u>average</u> number of arrows for the 25 runs at the end of the history.

The preceding model may or may not represent this situation. Let us leave that for a chemist or physicist or guppy breeder to decide.

Before we show our model to an expert, it might be well to consider possible objections he could reasonably be expected to raise. Hmmm— well, it does seem that the probability of a guppy leaving the cage should be greater when there are two guppies in the cage and only one outside than when the situation is reversed-doesn't it? Consider another possible model.

ANOTHER MODEL

You may easily discover a better simulation technique than the one suggested above. Be sure your next simulation takes into account that when the cage contains two guppies, then the next movement is twice as apt to be from the cage to the aquarium, as from the aquarium to the cage.

If you have trouble setting up your simulation model, here is an idea. Use three counters, one to represent each fish. If a given fish is in the cage, the corresponding counter will contain a 1, otherwise it will contain a zero. Two possible histories are:

(1) Fish #1 → 0 → 1 → 1 → 0 → 1 → 1 end

Fish #2 → 0 → 0 → 0 → 0 → 0 → 1 end

Fish #3 → 0 → 0 → 1 → 1 → 1 → 1 end

which could be represented more compactly as

000 → 100 → 101 → 001 → 101 → 111   end.

Another possible history could be

(2) Fish #1   0 → 0 → 1 → 1 end

Fish #2   0 → 1 → 1 → 1 end

Fish #3   0 → 0 → 0 → 1 end

which could be represented as

000 → 010 → 110 → 111   end.

Before you continue, write a program or flowchart to simulate this model.

In this simulation all three fish start outside of the transfer cage, i.e., F(1) = 0, F(2) = 0, F(3) = 0, THEN the computer generates a random number to determine which of the three guppies shall be the next one to change places (swim in if out, swim out if in cage). All three are in the cage when T = F(1) + F(2) + F(3) = 3.

The following flowchart summarizes the proposed program

```
              ┌─────────┐
              │  START  │
              └─────────┘
                   │
                   ▼
        ┌──────────────────────┐
        │      Initialize      │
        │                      │
        │   F(1)=F(2)=F(3)=∅   │
        └──────────────────────┘
                   │
                   ▼
        ╱─────────────────────╱
        │  PRINT out digits   │
        ╱─────────────────────╱
                   │
                   ▼
        ┌──────────────────────┐
        │    Select random     │
        │    number 1,2,3 to   │
        │  determine which of  │
        │   the three digits   │
        │      to change       │
        └──────────────────────┘
                   │
                   ▼
        ┌──────────────────────┐
        │ Change selected digit│
        │        ∅ → 1         │
        │        1 → ∅         │
        └──────────────────────┘
                   │
                   ▼
        ╱──────────────────────────╱
        │  PRINT out current digits │
        ╱──────────────────────────╱
                   │
                   ▼
                  ╱ Is ╲
                 ╱ sum  ╲
      No ◄──────◄ F(1)+F(2)+F(3) ►──── Yes
                 ╲  =3  ╱              │
                  ╲  ? ╱               ▼
                                   ╲ Stop ╱
```
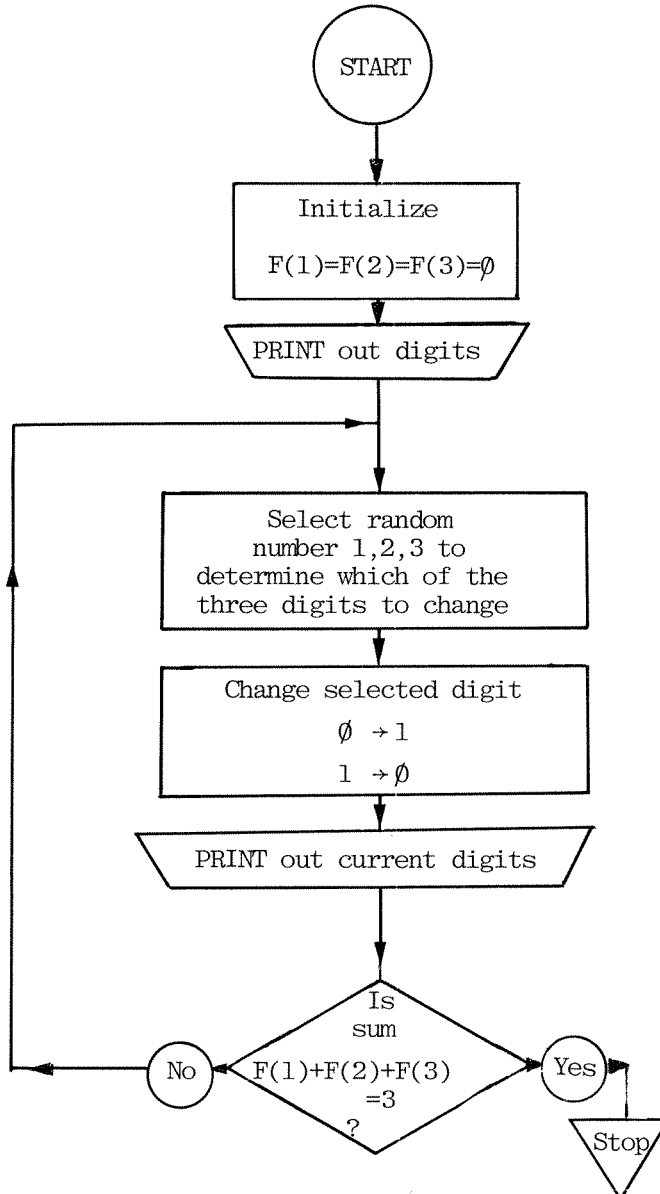
Here is one possible program for the previous flowchart – but yours may
be just as good, or even better than this one, so don't change yours –
not yet, at least.

<div align="center">GUPPY – SECOND MODEL</div>

```
1Ø    REM   GUPPY SECOND MODEL SIMULATION
2Ø    DIM F(3)
5Ø    REM     SET F1 = F2 = F3 = Ø  AS ALL GUPPIES ARE OUTSIDE CAGE
6Ø    F(1) = Ø
7Ø    F(2) = Ø
8Ø    F(3) = Ø
9Ø    PRINT F(1) ; F(2) ;  F(3) ;  " -> " ;
1ØØ   R = RND(Ø)
11Ø   IF R > .667 THEN 3ØØ
12Ø   IF R > .333 THEN 2ØØ
13Ø   REM    HERE ONLY IF F1 IS TO MOVE
14Ø   N = 1
15Ø   GOTO 5ØØ
2ØØ   REM    HERE ONLY IF F2 IS TO MOVE
21Ø   N = 2
23Ø   GOTO 5ØØ
3ØØ   REM    HERE ONLY IF F3 IS TO MOVE
31Ø   N = 3
5ØØ   REM    NOW READY TO MOVE FISH #N
51Ø   IF F(N) = Ø THEN 55Ø
52Ø   IF F(N) = 1 THEN 6ØØ
53Ø   PRINT "SOMETHING IS WRONG. F(" ; N ;") EQUALS"; F(N)
54Ø   STOP
55Ø   REM    MOVE F(N) INTO CAGE
56Ø   F(N) = 1
57Ø   IF F(1) + F(2) + F(3) <  3 THEN 9Ø
58Ø   PRINT F(1) ; F(2) ; F(3) ; "END--ALL IN CAGE NOW."
59Ø   PRINT : PRINT
595   GOTO 6Ø
6ØØ   REM    MOVE F(N) OUT OF CAGE
61Ø   F(N) = Ø
62Ø   GOTO 9Ø
```

Try it out. Debug the program and when it is running smoothly include it
in a program to run the simulation 25 times and print out the average
number of arrows for the 25 runs, at the end of the histories.

PRACTICE SET 9

1. Simulate the roll of a pair of dice. After your simulation is up and
running, enclose it in an over-program that will roll the pair of dice
1000 times and keep track of how many times the sum shown is 2, 3, 4,...,
12. Compare the output of your simulation with the theoretical outcome,
which you may either compute or discover in a book on probability, dice
or gambling.

2. Modify the program of Problem 1 so it assumes dishonest dice have been used in which the ⚁ is replaced with a second ⚃ .

3. Modify the program of Example 9-1, so the number of runs can be specified by the user.

4. Enclose the program of Example 9-2 in a FOR...NEXT loop that will permit you to run the experiment 500 times and tabulate a summary of the data obtained.

5. Explain the purpose of instructions 425 and 428 in the program of Example 9-2. Did the given program ever use $M(7)$? Why do you think the author left it so?

6. Create a flow chart for your final revised version of Example 9-2 (Problem 4) and use it to explain what your program does to a friend.

7. A batch of 80 ounces of plastic contains 100 small metallic impurities. Assume that the plastic is well mixed and that the impurities are distributed at random throughout the plastic when it is molded into 160 half-ounce replacement heart valves. If a valve contains one or more impurities it must be discarded. It is quite possible that anywhere from 60 to 159 metal-free valves may be obtained from the given batch. Write a computer program to simulate one hundred 80-ounce runs and determine the average number of metal-free heart valves you would expect on the basis of your simulation.

8. Extend the program of Problem 7 slightly so that in addition to giving you the average of the hundred runs, it will also keep track of the largest and the smallest number of metal-free valves that were manufactured from a single batch of the hundred runs.

9. Assume that in Shire approximately 100 Hobbits die in a given 30-day period. Write a computer simulation that will print out a table of twenty lines of output data, each line representing a 30-day period with 100 random deaths distributed among the 30 days. Print a reasonably clear header before the 20 lines of output data, since the Shire Health Department wishes to use your data for comparison with actual death rates.

10. A well-known and interesting problem asks the question, "In a set of thirty persons selected at random, what is the probability that at least two of the thirty persons have the same birthday (anniversary date) (i.e., same month and date of birth, but not necessarily born in the same year)?" How would you simulate this problem?

11. Write a program to simulate Problem 10 assuming 365 days per year. After debugging your central program, expand it so you run your simulation for 50 sets of 30 persons before you print out the approximate simulated experimental probability. Compare your simulated result with that computed. (See "the birthday problem" in a text on probability, if you need help.) Open Who's Who to some page and take the first 30 birthdays you find for a similar experiment.

12.   Simulate some simple game of your own choosing and have the computer summarize the results.   Suitable games might be "matching pennies", "put and take", or some very simple dice game but not anything as complicated as "Yahtzee" or "craps".

13.   Expand your program for Example 9-1 or Example 9-2 to run 1000 times and compute standard deviation as well as mean of the numbers involved.

14.   Debug the first guppy model.   After it is debugged and running, include it in an "over-program" that will run the model 25 times, printing a history each time and give the average number of arrows per run at the end of 25 runs.   If your runs all produce the same history, there is apt to be something wrong with the way you invoked the random number generator.   Fix it up and rerun the program.

15.   Debug the second guppy model.   After it is debugged (and checked that it doesn't always give the same history) incorporate your debugged program into a program that will run the model 25 times, print a history each time and give the average number of arrows per run.   Compare the results of model 1 and model 2.   Which do you feel is a better representation of the problem?   Why?

16.   Write a program to simulate a penny toss between two persons if the first person starts with P1 pennies and the second with P2 pennies, where P1 and P2 are input for the program.   Your program should print out the number of pennies each person has after each toss and stop as soon as either person has lost all his pennies.

17.   Write a program to simulate a ping-pong game with the initial input being two numbers (P1 and P2) which represent the probability of player 1 and player 2 returning a given shot successfully.   Change your model a bit to make it more realistic if you can.

18.   Same as Problem 17, but for the game of tennis.

19.   If you were to write a simulation of a baseball game, what variables would you wish to be able to read in and thus vary from run to run? Discuss a possible simulation with a colleague.   Make a rough flow chart of your proposed simulation.

20.   The dice game known as "craps" is played by noting the total number of points, when two dice are rolled, as follows:

    1.   If, on the first roll, you roll 7 or 11, you win.
    2.   If, on the first roll, you roll 2, 3, or 12, you lose.
    3.   If, on the first roll, you roll a 4, 5, 6, 8, 9, or 10, this number becomes your "point" and you continue rolling the dice until you either roll your "point" again or you roll a 7.   If you roll your point again before you roll a 7, you win.   If a total of 7 turns up first, you lose.

Use the computer to simulate a craps game.   Have the output show, on one line, the actual history as well as the first win or loss of the game.

21. Rearrange your program of Problem 20 so that you can simulate the game of "craps" under the assumption that each die carries the spots 1-5-3-4-5-6, instead of the usual arrangement.

22. Write a program to simulate the deal of 52 playing cards with NC cards per player and NP players. Your program should read in the values for NC and NP and check that NC*NP ≤ 52 before starting to deal. One way to deal using RANDU is to assume there are 52 cards numbered from 1 to 52 inclusive. You may wish to use DIMENSION C(52) and set the value of each C(I)=0 at the start, then once that "card I" is dealt, change the value of C(I) to +1 so that you do not deal the same card twice. (This causes hard feelings!) You may wish to store the hands in an array DEAL(NC,NP) by storing the numbers I, but in printing out the results, you should translate the stored integers into the usual suit and value designation. Use your program to deal 5-card poker hands to 8 players and also to deal 13-card bridge hands to four players.

LESSON 10
Types of Variables
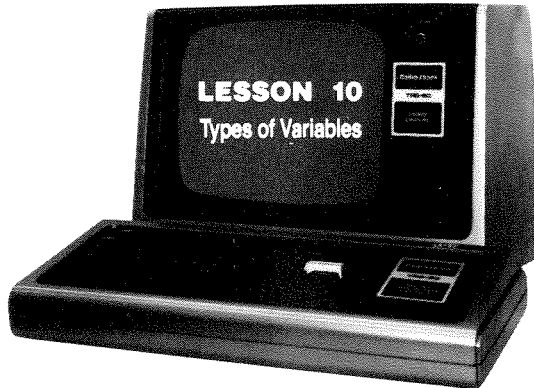
VARIABLE TYPES

BASIC uses four distinct data types for its variables. Each variable may be forced to be any of the four types, either by using DEF statements or by the use of special type designators described below. The four types are:

Single Precision Floating-point (6 digits of accuracy)
Double Precision Floating-point (16 digits of accuracy)
Integer (between -32768 and +32767)
String (up to 255 alphabetic or numeric characters)

The first three types are used for numerical computation and storage. String variables are never used in arithmetic. They permit the storage, examination and comparison of sequences of letters, numbers, blanks and special characters.

a) Single Precision Floating-point Variables   DEFSNG A,B,W-Z

Single precision floating-point numbers are the type we have been using for most of our computation. It is the option which the TRS-80 automatically elects for you unless you instruct it to do differently. Such computation is carried out to (the binary equivalent of) 7 decimal digits of precision inside of the computer, but only 6 digits of precision are printed on the CRT screen.

Since single precision is the automatic default condition, it is not necessary to use DEFSNG unless you need to change the precision of variables previously defined as INTEGER or DOUBLE PRECISION.

b) Double Precision Variables

You are already aware (See Lesson 5.) that it is possible to

obtain variables of greater accuracy (16 digits instead of 6 digits) for a variable by inserting

    DEFDBL X,Y

near the beginning of the program. This causes <u>every variable</u> beginning with the letter X or the letter Y to be double precision (17 digits internally, 16 digits displayed). It is also possible to indicate a range of initial letters in a DEFDBL statement. For example, DEFDBL A,C,S-W will cause every variable beginning with any of the variables A,C,S,T,U,V, W to be double precision unless it carries a special designator such as ! or % (see below).

c) <u>Integer Variables</u>

    It is also possible to specify that some variables are of <u>integer</u> form rather than the usual floating-point numbers. On short programs it is not worth the effort to do so, but long-running programs frequently can be speeded up noticeably by inserting

    DEFINT K,L,B-E

early in the program for those variables you know will always be integers in the range from -32768 to +32767. The DEFINT statement given above forces every variable beginning with K,L,B,C,D,E to be integer form. One of the most important savings is to make integers of the variables used in FOR...NEXT loops when appropriate. (Obviously, they must not take on fractional values in the step size.)

d) <u>String Variables</u>

    It is also possible on your TRS-80 LEVEL II BASIC to define a <u>string variable</u> (indicated by annexing a $ to any variable name) used to store strings (sequences) of letters and other characters up to 255 characters per string.

        Examples A$, C$, GR$, P1$, P2$

You may also define regular variables as string variables by using DEFSTR but be careful in doing this because if you designate DEFSTR A,D-G then every variable beginning with A,D,E,F,G will be a string variable and as such may <u>not</u> be used in arithmetic. Your authors usually prefer to use the annexed $ to indicate a string variable.

Try this little program:

        NEW   ENTER
    1Ø   CLS
    10Ø   INPUT "PLEASE TYPE YOUR NAME" ; N$
    11Ø   PRINT : PRINT : PRINT : PRINT

> N$ _

$ TELLS ME TO KEEP A SEQUENCE (STRING) THAT WONT BE USED FOR ARITHMETIC.

148
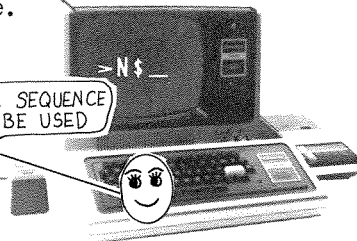
```
120   PRINT "HELLO"; N$; "." : PRINT "I HOPE YOU ARE ENJOYING THIS COURSE."
130   GOTO 110
      RUN   ENTER
```

Try it out on your computer. Depress   BREAK   to stop the program.

The output is a bit fast to read isn't it? To slow down the program, make it count from 1 to 500 after it prints by inserting

```
125   FOR Q = 1 TO 500 : NEXT Q
```

RUN it again.

Try several different versions of your name.
Try inserting your social security number or phone number instead of your name.
String variables and string instruction are discussed further in Lesson 14.

e)   Individual Variable Type Designators

It is also possible to make particular variables into integer variables by annexing a % sign after it — without changing other variables that begin with the same letter. For example

```
100   FOR X% = 1 TO 50
110      FOR K% = 1 TO 100
120         PRINT K%;
130      NEXT K%
140   NEXT X%
```

will run faster than

```
100   FOR X = 1 TO 50
110      FOR K = 1 TO 100
120         PRINT K;
130      NEXT K
140   NEXT X
```

The first program runs in about half the time of the second program.

It is possible to have particular variables as regular single precision floating-point variables (6 digit precision) by annexing a ! to double precision floating-point variables (16 digit precision). Likewise, it is possible to have particular variables as double precision floating-point variables by annexing a # to any standard variable name. The type designator (% = integer, ! = single precision, # = double precision) takes precedence over an earlier DEF statement.

149

Variable Type	Annexed Character	Examples	Typical Variables To Be Stored
Integer Variable	%	A%,X%,K%,NI%	-2746,-5,0,1,17,3000
Single precision (6 digit) variable	!	A!,BX!,XI!	4,-50.73,.1276,1.23456E+09
Double precision (16 digit) variable	#	A#,XI#,C#,BV#	1.234567887654321D+15

Note: If you wish to enter or use double precision constants, you can save a lot of trouble by placing a D at the end of it. Thus A# = 1234.7D. Also, use D in place of E to indicate the exponent of 10. Thus:

    A = 6.02486E+23
    A# = 6.024859316852145D+23

Note that B, B%, B!, B# and B$ may all be used in the same program and each is a name for a different variable.

SUBSCRIPTED VARIABLES

The statement

DIM X(15), Y(15)

permits you to have sixteen different X-values

X(0), X(1), X(2), X(3), X(4),..., X(14), X(15)

and sixteen different (associated?) Y-values

Y(0), Y(1), Y(2),...,Y(14), Y(15)

in the computer at the same time .

Note that
X(3) and X3 are different variables, and each is different from X.

    Such "subscripted variables" are important in statistics, where it is common to have sets of related data, say 25 temperatures and the 25 related product yields. The following program permits you to input 25 sets (T,Y) of temperature-yield data.

    1Ø   DIM T(25), Y(25)
    1ØØ   FOR K = 1 TO 25
    11Ø   INPUT T(K), Y(K)
    15Ø   NEXT K

Note that we have not used location T(Ø) nor Y(Ø), but we could have.

The program might be expanded to also compute the average T value and the average Y value by accumulating the sum of the T's and the sum of the Y's. After the sums are computed, divide each by the number of data pairs (here 25) and store the result in $T(\emptyset)$ and $Y(\emptyset)$

```
1∅    DIM T(25), Y(25)
9∅    T = ∅ : Y = ∅
1∅∅   FOR K = 1 TO 25
11∅       INPUT T(K), Y(K)
12∅       T = T + T(K)
13∅       Y = Y + Y(K)
15∅   NEXT K
16∅   T(∅) = T/25
17∅   Y(∅) = Y/25
```

(Note: This is only a fragment of a program. It permits you to enter data into T( ) and Y( ), but does not produce or PRINT anything.)

Note that T and Y were used <u>inside</u> the FOR...NEXT loop rather than using $T(\emptyset)$ and $Y(\emptyset)$. This saves computer time. Most expert programmers consider it gauche to use a subscripted variable <u>whose</u> <u>subscript</u> <u>does</u> <u>not</u> <u>change</u> inside of a FOR...NEXT loop.

If desired, the program could be extended to compute standard deviation, correlation, mode, median, and other statistical measures. If you are familiar with statistics, you can extend the program yourself. You can even compute a "line of regression" or a "curve of regression" for the given data.

Our purpose was to introduce subscripted variables, a very handy tool for solving many problems.

ARRAYS

It is also possible, in Level II BASIC to store <u>arrays</u> (matrices) of numbers (or letters or symbols). Consider a set of class grades for 20 students on each of six tests. Although it is possible to store student names, it is simpler to use student numbers instead.

The 20 by 7 array is:

	Col 0	Col 1	Col 2	Col 3	Col 4	Col 5	Col 6
	Student ID no.	Test 1	Test 2	Test 3	Test 4	Test 5	Test 6
Row 1	12516	86	74	92	71	77	64
Row 2	12617	57	62	71	69	79	85
Row 3	77969	94	98	.	.	.	.

```
Row 4   38491       .         .         .         .       .        .

         .          .         .         .         .       .        .

         .          .         .         .         .       .        .

         .          .         .         .         .       .        .

Row 20  21201      87        93        91         .       .       92
```

We represent it in the computer by

DIM G(2∅,6)

This gives an array with 21 rows and 7 columns, for storage. We shall
not use row 0 for the storage data in this problem, but shall use column
0 to store the student's ID Number.

The 94 grade on test 1 of student number 3(ID = 77969) is located in po-
sition (3,1). Position (2,4) contains the grade of 69 for the 2nd student
(ID = 12617) on the 4th test.

The following program will read in the grades for the C-th test, to be
supplied by the teacher. For C = 0, the student ID# may be typed in, if
desired.

```
1     REM    PROGRAM TO INPUT QUIZ GRADES FOR CLASS OF 20 STUDENTS
1∅    DIM G(2∅,6)
1∅∅   INPUT "TYPE TEST NUMBER PLEASE"; C
11∅   FOR R = 1 TO 2∅
12∅      PRINT "TYPE GRADE ON TEST" ;C; "FOR STUDENT #"; G(R,∅),
13∅      INPUT G(R,C)
14∅   NEXT R
```

The following program will display the entire array of scores, as
well as computing and printing the average grade on the six tests for all
20 students. It assumes the data is already stored in the computer(from
tape, or by hand, or ??). This program may be combined with that above.
If no grades are entered, the computer will print zeros.

```
2     REM    PROGRAM TO DISPLAY AND AVERAGE STUDENT GRADES ALREADY IN THE
             COMPUTER
1∅    DIM G(2∅,6)
2∅∅   FOR R = 1 TO 2∅
21∅      T = ∅
22∅      PRINT  "STUDENT #" ;R, "I.D.#" ; G(R,∅),
23∅      FOR C = 1 TO 6
24∅         PRINT G(R,C);
25∅         T = T + G(R,C)
26∅      NEXT C
27∅      PRINT "->"; T/6
28∅   NEXT R
```

Examine the program to understand how it works. If you have any questions, ask your instructor.

Note the use of indented instruction inside each FOR...NEXT loop. This is not important as far as the computer is concerned, but is a very helpful practice to help humans to perceive the structure of your programs. Your author sincerely recommends the practice.

LESSON 11
Tips
and Error Messages
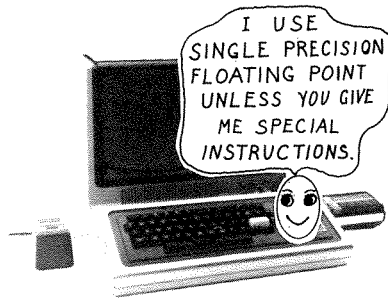
We have collected a few tips and cautions that you may never need, but that you still may find helpful.

TIPS

Don't let anyone spill beverage on or near your TRS-80. The only way to do this may be to forbid beverages near the computer. It is up to you, but a spilled Coke portends real grief forever afterward.

If you are plagued by double letters in your typing, it is because dust and dirt are forcing your TRS-80 to get multiple signals on a single key stroke. LEVEL II BASIC will have this problem more frequently than LEVEL I, since LEVEL II stores key strokes internally (great for fast typists). Keyboards manufactured in the 1980's seem to be less plagued with keyboard bounce than earlier TRS-80's. It is possible to remove the key tops and gently clean the contacts, but it is a lot easier to use the KEYBOARD DEBOUNCE (KBFIX) program furnished with LEVEL II TRS-80. It is a good idea to use a dust cover on the keyboard providing it is turned off (else heat may build up).

Begin your programs in line 1$\emptyset\emptyset$. Use lines 1 to 99 to identify yourself and program rather completely, as well as giving credit to any source you used before modifying your program.

If you would like your REM to stand out even more than it does by inserting 5 blanks after the REM, try depressing $\boxed{\downarrow}$ after typing REM and then type six or eight spaces before your REMark.

ERROR OS (out of string space) can frequently be remedied by inserting CLEAR n with a larger n value (say 1$\emptyset\emptyset$, 5$\emptyset\emptyset$, or MEM/4). The default value is CLEAR 5$\emptyset$ whenever you turn on the TRS-80 or type NEW $\boxed{\text{ENTER}}$.

If you use a BASIC program from a book or magazine, watch for RND(1) statements. Some BASIC's use RND(1) where the TRS-80 uses RND($\emptyset$), namely, to generate a random decimal between $\emptyset$ and 1.

154

If you type LLIST instead of LIST and do not have a line printer turned on, the TRS-80 will lock up. Open the small door at the back left of the keyboard unit and push the small round reset button.

The TRS-80 is designed for most room temperatures, but if you have clutter around it, that prevents air circulation. If you must use it in a hot place, put a couple of 1x1x5-inch blocks under the feet to permit extra air circulation.

Programs that produce a full screen of output can be ended with a tight loop (such as 9999 GOTO 9999) to prevent having break into the screen. Use [BREAK] to get out of a tight loop.

```
READY
>_
```

If your program seems allright, but you get an SN ERROR that you can't identify, you may have depressed the [SHIFT] key while typing that line. (I tend to do this on [@] .) Retype the line in question, watching your [SHIFT] key carefully.

If you are an electronics buff, by all means get the TRS-80 Microcomputer Technical Reference Handbook (1980, $9.95, Radio Shack). It has wiring diagrams you'll want to have. Remember, if you open the TRS-80 case, your guarantee is void.

Use CLOAD? to verify your Level II BASIC programs.

Double precision variables are not acceptable in FOR...NEXT loops.

If you have experienced problems with double precision variables not converting correctly (TRS-80 does not use decimal notation internally), it may be because you have not entered them correctly. If a double precision variable has fewer than 7 digits when typed, follow it with the letter D.

Run this to see what is meant.

```
100   S = .1
110   D# = .1
120   PRINT S,D#
```

Your output will look something like

```
        .1          .10000000149011161
```

If you change 110 to

```
110   D# = .1D
```

all will be well. Try it and see.

Static electricity can play havoc with your TRS-80 and cassettes. If low humidity and carpet friction produce static electricity shocks in your computer room, do something about it. Remove the carpets or put down a non-static-producing rubber or plastic pad and increase the humidity in the room, if possible.

Don't force the 110V-plug on your video display into a socket or extension cord that is not designed to preserve polarity. One blade on the video 110V-plug is larger than the other—in this way, the video display chasis is grounded through the power cord. If you use an extension cord or multiple-outlet plug that does not preserve this polarity, you can get into trouble.

No doubt you have already discovered the HORIZONTAL HOLD and VERTICAL HOLD controls located on the rear of your video display. Once set, it should not be necessary to readjust them very often.

The BRIGHTNESS (B) and CONTRAST (C) controls are located on the front bottom right of your video display. Keep each in about the middle of the range. To lengthen tube life, DO NOT ADJUST TO MAXIMUM BRIGHTNESS OR CONTRAST. It may burn your tube if the display remains fixed.

Turn OFF the video display whenever you leave the computer, even if you leave the computer running.

Electrical interference from an elevator or other motor, a faulty flourescent light or even the switch in a typewriter are not apt to interfere with your TRS-80 unless you have added disk to your system. If you are considering disk or other expansion interface items, you should add a couple of line filters at your power source as well.

SAVING MEMORY SPACE

If you have only 4K of memory, you may find that unwelcome OM ERROR message signifying all of the available memory has been used or reserved. There are several things you can do to conserve memory space.

Cut down on the size of any arrays used.

Use multiple statements per program line with a colon between statements.

Use integer variables whenever possible--in FOR K% = 3 TO 3ØØ, for example. Use the same variable in all FOR...NEXT loops that are not nested.

Delete unnecessary parentheses in arithmetic statements.

Cut down on both the number and length of string variables.

Delete unnecessary spaces in all statements.

Eliminate extra variables, reuse variables no longer needed, keep variable names short.

If necessary, also delete REM statements.

ERROR OM can sometimes be cured providing you are using no string variables by inserting CLEAR Ø at the beginning of your program.

OS ERROR frequently seems to be the result of a missing CLEAR *n* statement, where *n* is greater than 5Ø. Try CLEAR 2ØØ or CLEAR MEM/4.

SPEEDING UP PROGRAMS

Although computers are fast, they do run into time difficulties on long programs. Usually the solution is to think hard about the algorithm being used. It is frequently possible to devise a much faster algorithm once the problem is fully understood. There are also programming techniques, known to most experienced computists, that save considerable time.

Don't use subscripted variables inside of a FOR...NEXT loop unless the subscript is changing.

Combine several statements per line (separated by colons).

Use integer variables in FOR...NEXT loops that have integer step sizes.

Define the most commonly used variables first--this puts them near the top of the variable table and saves look-up time if many variables are involved.

Don't recompute values needed repeatedly; store them instead. (This is particularly vital inside of nested FOR...NEXT loops.)

Use small data sets and short FOR...NEXT loops in debugging long programs.

Use faster forms whenever possible.
B + B is faster than 2 * B.
.2 * C is faster than C/5. (Avoid division when possible.)
D * D * D * D is much faster than $D \uparrow 4$.

Don't recompute trigonometric or log functions for the same value.

To evaluate $Y = 5 \sin^4 X + 8 \sin^3 X - 2 \sin^2 X$

use S = SIN(X)

Y = ((5 * S + 8) * S - 2) * S * S.

Use POKE graphics. This can cut graphics time to 1/5 of SET(X,Y).

Use stored constants (a letter like B or P) rather than numeric constants in the statements that are executed frequently.

Probably the most important thing you can do to make programs run faster is to analyze which subpart of your program uses the greatest amount of time and then rewrite that portion as a subroutine in Z-80 machine language. Call the subroutine using the USR(X) routine. However, that requires learning another language. See Chapter 8 of your TRS-80 Level II BASIC Reference Manual under USR(X) and a text on Z-80 machine language. Radio Shack has a book, TRS-80 Assembly Language Programming by Wm. Barden, Jr., (Cat. #62-2006) which will be helpful if you are sufficiently interested to wish to learn Z-80 chip language.

ERROR MESSAGES

We have discussed, under EDITING, Lesson 8, the appearance on the screen of the message

?SN ERROR IN 1000

LEVEL II BASIC also has other messages that help you discover errors in a program. The following ERROR messages are among those you may encounter. (If you have a disk on your system—consult the appropriate reference manual for additional messages.)

EXPLANATION OF ERROR MESSAGES

BS Subscript out of Range: An attempt was made to assign a subscripted variable or a matrix element with a subscript beyond the DIMensioned range.

CN Can't Continue: A CONT was ENTER ed at a point where no continuable program exists, e.g., after program was ENDed or EDITed.

DD Redimensioned Array: An attempt was made to DIMension a matrix which had previously been dimensioned by DIM or by default statements. It is a good idea to put all dimension statements at the beginning of a program.

FC Illegal Function Call: An attempt was made to execute an operation using an illegal parameter. Examples: square root of a negative argument, negative matrix dimension, negative or zero LOG arguments, SET (X,Y) with one value out of bounds, etc. Or USR call without first POKEing the entry point.

FD Bad File Data: Data input from an external source (i.e., tape) was not correct or was in improper sequence, etc.

ID Illegal Direct: The use of INPUT as a direct command, without a statement number.

L3 DISK BASIC only: An attempt was made to use a statement or function which is available only when the TRS-80 Mini Disk is connected via the Expansion Interface.

LS String Too Long: A string variable was assigned a string value which exceeded 255 characters in length. Break it into two or more strings.

MO Missing Operand: An operation was attempted without providing one of the required operands.

NF NEXT without FOR: NEXT is used without a matching FOR statement. This error may also occur if NEXT *variable* statements are reversed in a nested loop.

NR No RESUME. End of program reached in error-trapping mode.

OD Out of Data. A READ or INPUT # statement was executed with insufficient data available. DATA statement may have been left out or all data may have been read from tape or DATA.

OM Out of Memory. All available memory has been used or reserved. This may occur with very large matrix dimensions, nested branches such as GOTO, GOSUB, and FOR...NEXT loops, or with large strings. Usually the program can be rewritten so that it uses less memory. See separate suggestions.

OS Out of String Space. The amount of string space allocated was exceeded. Usually this can be fixed by starting program with CLEAR n for some $n > 50$ or by using CLEAR MEM/2.

OV Overflow. A value input or derived is too large or small for the computer to handle.

RG RETURN without GOSUB. A RETURN statement was encountered before a matching GOSUB was executed.

RW RESUME without ERROR. A RESUME was encountered before ON ERROR GOTO was executed.

SN Syntax Error. This usually is the result of incorrect punctuation, open parenthesis, an illegal character or a misspelled command.

ST String Formula Too Complex. A string operation was too complex to handle. Break up operation into shorter steps.

TM Type Mismatch. An attempt was made to assign a non-string variable to a string or vice-versa.

UE Unprintable Error. An attempt was made to generate an error using an ERROR statement with an invalid code.

UL Undefined Line. An attempt was made to refer or branch to a nonexistent line.

/∅ Division by Zero. An attempt was made to use a value of zero in the denominator.

TRS-80 Level II BASIC has an unusual extra "goodie" you may find handy. Commands such as RUN, NEW, LIST, DELETE 100-499, and similar keyboard commands can be inserted into and executed from a program. Try the following programs to get the feel of the idea.

```
100 CLS
110 FOR K=1 TO 30
120    PRINT K;
130 NEXT K
140 PRINT
RUN
```

Then add

```
150 RUN
```

and RUN it again.

Now, change 150 to

```
150 LIST
```

and RUN it again.

Depress BREAK and change 150 to

```
150 DELETE 100
```

LIST the program. RUN it, then LIST it again. Note that 100 has disappeared. Change 150 to 150 NEW
and RUN it again.

It seems rather straightforward until you discover that if you reenter the program 100 to 140 and add

```
150    LIST
160    RUN
```

the program never gets to 160. Most commands (RUN is an exception) send the computer back to READY mode instead of continuing the program. You can overcome this by using

```
PRINT @ 970, "PLEASE TYPE  RUN (ENTER)."
```

just before the DELETE or LIST command.

Try the program on the following page.

```
1Ø9 CLS
11Ø FOR K=1 TO 3Ø
12Ø    PRINT K;
13Ø NEXT K
14Ø PRINT : PRINT
15Ø PRINT "PLEASE TYPE    RUN (ENTER)"
16Ø DELETE 1Ø9-16Ø
165 PRINT : PRINT : PRINT
17Ø PRINT "WHEN THIS IS LISTED NOW, LINES 1Ø9 TO 16Ø WILL BE MISSING."
18Ø PRINT : PRINT
19Ø LIST
RUN  ENTER
```

    Advanced programmers use this technique to get extra space when
OS ERRORS plague them. Early parts of the program can be deleted before
the string variables are used. A sample might be:

```
1Ø
  .
  .
  .
  .
1ØØ
  o
  .
  .
  .
499
5Ø4 REM    TRANSITION TO GET MORE STRING SPACE
5Ø5 CLS
51Ø PRINT : PRINT
515 PRINT "PLEASE TYPE    RUN(ENTER)"
52Ø DELETE 1Ø-499
6ØØ REM    MAIN PROGRAM USING STRINGS STARTS HERE
61Ø CLEAR 5ØØØ : DIM B$(4Ø), M$(5Ø)
62Ø PRINT  "THIS IS THE MAIN STRING PROGRAM."
7ØØ LIST
```

LESSON 12
Extended
Print Instructions

You are already familiar with the instruction

PRINT

which gives a blank line and

PRINT A,B,C,D

which gives wide (4 column) spacing where a comma is used and close-packed spacing where a semicolon is used.

PRINT A; B,C,D

is also familiar (see Lesson 2).

BASIC has several other useful instructions including

```
PRINT   TAB( )
PRINT   USING A$,K
PRINT   STRING$(K,"#")
```

PRINT TAB (*expression*)

The "*expression*" may be a constant or a variable or a computed expression which is an integer between Ø and 255. This "tabs" the cursor to the indicated position. It is <u>not</u> possible to move the cursor to the left by tabbing. If the cursor is already to the right of the TAB expression, the TAB is ignored. Any indicated printing occurs at the place where the cursor is.
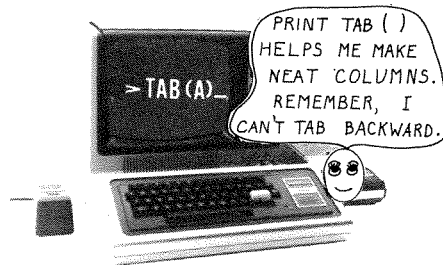
Try the following:

1ØØ A = 3

11Ø B = 5

12Ø PRINT TAB(A) A; TAB(B) B; TAB(A+B) "A+B"; TAB(35) "35"; TAB(1Ø*B+A) "Z"

on your TRS-80.

The TAB(   ) function is particularly useful in graphing and the creation of special tables and forms.

PRINT TAB ( )
HELPS ME MAKE
NEAT COLUMNS.
REMEMBER, I
CAN'T TAB BACKWARD.

It is possible to format the output in a more precise manner than is available on the ordinary PRINT instruction employing ; , and " " or even the PRINT @ instruction.

The PRINT USING instruction makes this possible. Try the following program.

```
10    CLS
900   PRINT  "PLEASE TYPE YOUR NAME, INCLUDING MIDDLE INITIAL"
910   INPUT N$
1000  FOR K = 917 TO 1130 STEP 13.25
1010     PRINT  "PAY TO THE ORDER OF  "; N$; "  ";
1020     PRINT USING "**$###.## DOLLARS"; K
1050  NEXT K
```

In instruction 1020, the "**$###.## DOLLARS" gives the format in which the material is to be printed. The **$ says to fill up any space before the first digit with asterisks and then a $. The decimal point followed by two #-signs (.##) indicates exactly two places beyond the decimal point are to be printed. Note the change in number of * where the amount changes from $996.50 to $1009.75 in the output when you run this program. Consult your TRS-80 Level II BASIC Reference Manual, Chapter 3 for additional information.

PRINT USING statements may use any of the following field specifiers:

Numeric Character	Function	Example
#	Numeric field (one digit per #).	###
.	Decimal point position.	##.###
+	Print leading or trailing sign (plus for positive numbers, minus for negative numbers).	+#.### #.###+ -#.### #.###-
-	Print trailing sign only if value printed is negative.	###.##-
**	Fill leading blanks with asterisk.	**###.##

(Continued next page)

165

Numeric Character	Function	Example *(continued)*
$$	Place dollar sign immediately to left of leading digit.	$$####.##
**$	Dollar sign to left of leading digit and fill leading blanks with asterisks.	**$####.##
↑↑↑↑	Exponential format, with one significant digit to left of decimal.	#.######↑↑↑↑

String Character	Function	Example
!	Single Character	!
%spaces%	String with length equal to 2 plus number of spaces between % symbols.	% %

PRINT STRING$ (K,"CHARACTER" or NUMBER)

This is very useful in graphing since it returns a string of K characters each of which is the character enclosed in quotation marks. It is also possible to use the ACSII number associated with a given character. The value K must be between ∅ and 255, as must any number used for ASCII characters. See Lesson 4 for list of ASCII equivalents.

PRINT STRING$ (25, "#") produces

####################### (25 of them)

on your screen.

The following program plots the number K followed by a string of K^2 symbols:

```
1∅∅  CLS
11∅  FOR K = 1 TO 1∅
12∅     L = K*K
13∅     PRINT K; STRING$(L,"%")
135     PRINT
14∅  NEXT K
15∅  GOTO 15∅
```

Try it on your screen and see what happens. Then change the symbol inside the "   " to some other symbol of your choice.

Lesson 4 included a program to generate a bar graph (histogram).

```
  5  CLEAR 2∅∅
 1∅  CLS
1∅∅  PRINT @ 6, "GRAPH OF ENROLLMENTS AT THE UNIVERSITY OF OKLAHOMA"
11∅  READ Y,D
115  IF Y < ∅ THEN 115
12∅  DATA 1940,21,1950,28,1960,32,1970,51,1980,59,-1,-1
13∅  PRINT Y; STRING$(D,143)
14∅  GOTO 11∅
```

The instruction
   5 CLEAR 2∅∅ clears all variables and sets aside 200 bytes for string
               variable use. This is maybe more than we need.

11∅ READ Y,D reads two values from the DATA string.

Instruction

13∅ PRINT Y; STRING$(D,143) prints the current value of Y (read in 11∅
     from DATA) and follows this with a D-long string of the symbol
     having ASCII equivalent of 143 which is

167

The same bar graph results as in the Lesson 4 program using SET (X,Y):
SET (X,Y+1), but in much less time and with a shorter program. The DATA
statement is also easy to change in the new program.

Note that if your D value is greater than 59, the bar will be continued
on a second line. However, D may be as great as 256 if desired, but
you'll need to use CLEAR 250 if D is that large.

Here is another program that may interest you. Try it out your-
self.

```
100  CLEAR 63
110  CLS
120  FOR Y = 0 TO 960 STEP 64
130     PRINT @ Y, STRING$(63,191)
140  NEXT Y
150  GOTO 150
```

After you see what it does, run the same program with 100 changed to

100 CLEAR 60   *(Why did you get OS ERROR, if you did?)*

You may also wish to change to 100 CLEAR 100 and then change STRING$
(63,191) to STRING$(64,191).   See if you can explain the unexpected
dark band at the bottom of the screen when you RUN it.

Sometimes it is handy to be able to clear only a portion of the
screen. Try this:

```
109 CLS
110 FOR K=1 TO 205
120     PRINT K;
130 NEXT K
140 PRINT
200 PRINT  @  256, CHR$(30); "THIS IS THE NEW LINE FROM STMT 200.";
300 PRINT  @  512, CHR$(30); "THIS CAME FROM STATEMENT 300.";
500 GOTO 500
RUN
```
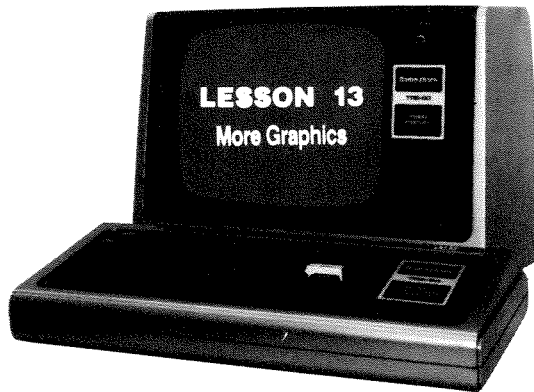
Depress BREAK to regain control.

After you have RUN this program a couple of time with the semi-
colon at the ends of lines 200 and 300, remove one of the semicolons and
RUN the revised program.
Investigate what happens if you insert:

299 PRINT CHR$(23);

You may also wish to use CHR$(31) in statement 300 to see what happens.

LESSON 13
More Graphics

More Graphics

Lesson 4 provided an introduction to graphics that may be adequate for your needs. However, you should know that the TRS-80 has many graphic abilities in addition to the

CLS	clears screen
SET (X,Y)	turns on rectangular spot at (X,Y)
RESET (X,Y)	turns off rectangular spot at (X,Y)
POINT (X,Y)	tests to see if point (X,Y) is turned on, if so it returns a logical TRUE (-1), otherwise a logical FALSE ($\emptyset$). Can be used in IF instructions (See Lesson 14, for details.) on logical operators.
CHR$(N)	produces actions, letter or symbol that corresponds to code N where $0 \le N \le 255$

For example: the operator ASC(*symbol*) is the inverse of CHR$(N). It produces the numeric (ASCII) code corresponding to the symbol in paren-theses (the first symbol in the string, if the parentheses contain a string of more than one symbol). This is used in an excellent cipher program considered in Lesson 14 of this text.

In Lesson 4 you ran the following program:

```
1Ø   CLS
1ØØ  FOR K = 33 to 191
11Ø     PRINT K,
12Ø     FOR L = 1 to 45
13Ø        PRINT CHR$(K);
14Ø     NEXT L
15Ø     PRINT
17Ø  NEXT K
2ØØ  GOTO 1ØØ
```

If your TRS-80 shows both upper and lower case letters (a modification you may obtain, if needed) CHR$(K) for K=65 to 90 will display upper case letters A to Z while K=96 to 127 displays the corresponding lower case (SHIFT) symbols corresponding to @, A to Z, ↑, ↓, ←, →, — . Most TRS-80's display only upper case letters for both ranges.

169

Run it again now. Change instruction 1∅∅ to

       1∅∅ FOR K = 2∅ TO 255

and run it again, after turning to the table given in Lesson 4 that
discusses the effect of CHR$(N) for various N values.

The pixel-blocks that appear corresponding to

       PRINT CHR$(K)

for K between 129 and 191 are particularly useful in speeding up graphics.
At first glance they may seem haphazard, but actually the choice of num-
bers associated with the graphic you wish to light up is both logical and
easy to use, once it is explained. Each large block (letter & space be-
tween lines) is divided into six small rectangles (pixels)
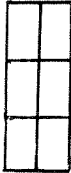
The individual pixels are numbered.

1	2
4	8
16	32

To light up the pixels numbered 1, 2, 8 and 16,
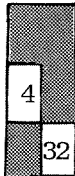
add the sum of pixels used: 1 + 2 + 8 + 16 = 27

and add that sum to 128 giving

       128 + (1 + 2 + 8 + 16) = 155.

The instruction

       PRINT CHR$ (155)

will light up the   desired pixels much faster than

will SET (X,Y).

170

Much time can be saved in graphics by using CHR$( ).
You may use the instruction

POKE *address, pixel number*

where *address* is 15360 plus the number between 0 and 1023 which locates.
the position (see Lesson Four) of the 6-block under consideration.
The *pixel number* is the number (128 + (sum of desired pixels)).

For example:

The entire 6-block

1	2
4	8
16	32

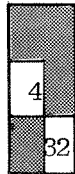has sum 63 so 128 + 63 = 191 is

the pixel number to turn on the entire 6-block.

The program

```
100   CLS
110   FOR X = 15360 TO 16383
120       POKE X, 191                    (See Lesson 16 for POKE instruction.)
130   NEXT X
140   GOTO 140
```

RUN

will "white out" the entire screen in record time. It can be combined
with:

```
140   X = 2 + RND(121) : Y = 1 + RND(43)
150   RESET (X,Y) : RESET(125-X,Y)
160   RESET (X,46-Y) : RESET(125-X,46-Y)
170   GOTO 140
```

to produce a reverse art pattern, if desired.

Try some additional experiments with the above 100-140 program such
as combining it with

```
140   PRINT @ RND(1000), "HI your name";
150   IF RND (0) >.01 THEN 140 ELSE 100
```

or some other experiments of your own design. Remember as long as you
keep beverages away from the TRS-80 and do not abuse it, you probably will
not harm it by your programming experiments -- and the best way to learn
computing is to compute and then to think about the results before
continuing.

If you need to insert a variable number of blanks in a line

PRINT CHR$ (192+k)

            will insert k blanks for k = 0, 1, 2,...,63.

This may be used to good effect in graphing by using

        PRINT CHR$ (192 + X); "+"

as in the following program to graph a portion of $X = .062Y^2$ which lies
in the first quadrant.

104   PRINT "GRAPH OF X = .Ø62*Y*Y"
11Ø   Y = 3Ø
12Ø   X = INT(.Ø62*Y*Y)
13Ø   PRINT X; CHR$(192 + X); "+"
15Ø   Y = Y -2
16Ø   IF X >2 THEN 12Ø ELSE 16Ø

Note that it will be necessary to depress |BREAK| to get the computer's
attention after the graph is plotted.(WHY)?

If we had not created the "tight loop" by using ELSE 16Ø, then when X
got to be < = 2 the display screen would have inserted

READY
>—

onto the screen and scrolled the top off the screen. If you are not sure
of this,

change 16Ø to 16Ø IF X >2 THEN 12Ø

and reRUN the revised program.
    The second edition of the LEVEL II BASIC Reference Manual (appendix C/6)
shows a map of the graphic pixel blocks and their corresponding numbers. Both
the character spacing (16 lines of 64 characters)numbered from 0 to 1023
and the pixel locations for use with SET (X,Y) (48 lines with $0 \leq Y \leq 47$ of
128 pixels with $0 \leq X \leq 127$) are shown mapped onto the grid. In each case
the numbering starts in the upper left corner with Ø or (Ø,Ø).

    If you remember the diagram on the right
and use 128 + (pixels to be lighted), you
will not need the reference.

1	2
4	8
16	32

LESSON 14
String and Logical
Operators
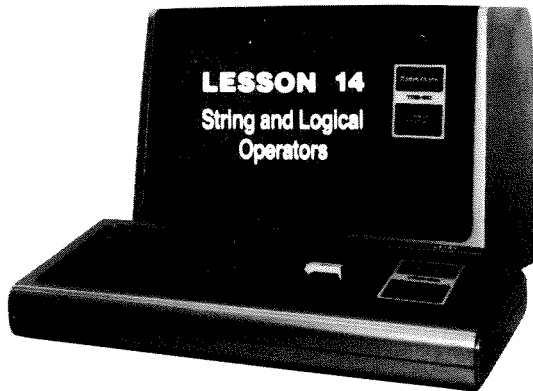
STRING FUNCTIONS

Let's re-examine the program of Lesson 4.

```
1Ø    CLS
1ØØ   INPUT "PLEASE TYPE YOUR NAME"; N$
11Ø   PRINT:PRINT:PRINT
12Ø   PRINT "HELLO ";N$;". ": PRINT"      I HOPE YOU ARE ENJOYING THIS
                                                         COURSE."

125   FOR Q = 1 to 5ØØ:NEXT Q
127   PRINT CHR$(23)
13Ø   GOTO 11Ø
```

Line 1Ø clears the screen.
Line 1ØØ prints what is in quotes and then awaits your input to string variable N$.
Line 11Ø prints 3 blank lines (i.e., moves the next output line down
   3   rows).
Line 12Ø prints HELLO, followed by whatever you typed into N$, and then on the next line prints

     I HOPE YOU ARE ENJOYING THIS COURSE. with a nine-space indentation, if you allowed nine spaces.
Line 125 is a "timewaster". It forces the computer to count from 1 to 5ØØ by ones before continuing.
Line 127 changes all output to double-size until the next CLS instruction is executed. This eliminates half of the material currently on the screen, but new material will be printed in double-wide characters.
Line 13Ø sends the program back to line 11Ø to print three more blank lines followed by HELLO *your name* (double-size this time) etc., etc.,.....
until you depress the BREAK key to stop it.

That wasn't too hard was it?

Play around a bit with the messages in line 12Ø and change them to suit your whim—be sure to include N$ as part of your output—note that N$ is a variable and must be outside of the quotes, separated from the quotes by semicolons.

String variables use up quite a lot of memory space—not as much as on many computers, since the TRS-80 LEVEL II string variable uses only as much space as it needs—i.e. short strings use less memory than long strings. On many computers, string variables require the same (maximum) space even if not all is needed. If you have a 4-K Level II, you may run out of memory if you use many variable strings. The following program is designed to create very simplistic sentences and runs on 4-K. Note that string variables may be dimensioned like floating-point variables.

```
2     CLEAR 1ØØØ
5     CLS
1Ø    DIM N$(7), V$(2)
1ØØ   PRINT "PLEASE TYPE 8 NOUNS."
11Ø   FOR K = Ø to 7
12Ø       PRINT K + 1, :INPUT N$(K)
13Ø   NEXT K
2ØØ   PRINT "TYPE 3 VERBS.  PLEASE USE DIFFERENT TENSES."
21Ø   INPUT "1 ";V$(Ø)
22Ø   INPUT "2 ";V$(1)
23Ø   INPUT "3 ";V$(2)
5ØØ   PRINT "THAT IS FINE.  NOW I'LL CREATE SOME SIMPLE SENTENCES."
6ØØ   PRINT N$(RND(8)-1);" " ; V$(RND(3)-1);" ";N$(RND(8)-1);"."
62Ø   GOTO 6ØØ
```

Put it on your computer and RUN it. If you should run out of memory (indicated by OM ERROR or OS ERROR) see if you can figure out what to do about it (see Lesson 11).

It is also feasible to store the words in DATA statements. In this case they are read into the program using a READ statement as the following program, designed with tongue-in-cheek to generate impressive phrases for use in reports and grant requests.

READ W$(I) is like INPUT W$(I), except that it obtains the string value for W$(I) from a DATA statement instead of from the keyboard.

The individual string entries in the data statement are each enclosed in quotes and entries are separated by commas. The computer will go to the next DATA statement if the DATA statement on which it is working is exhausted. DATA may also read from tape—but not here.

DATA statements may also be used to store numerical data. In this case numerical variables (no $) are used in the READ statement and quotes are not used in the DATA statement.

```
1∅   DIM W$(35)
2∅   FOR I = ∅ TO 35
3∅      READ W$(I)
4∅   NEXT I
5∅   DATA "FUNDAMENTAL", "BASIC", "INTUITIVE", "STUDENT-CENTERED",
     "HOMOGENEOUS", "MODULAR"
51   DATA "PARALLEL", "CENTRAL", "EQUAL", "SUPERIOR", "COLLECTIVE",
     "JUDGMENTAL"
52   DATA "ACCOUNTABLE", "UNGRADED" , "NON-SEXIST", "BEHAVORIAL",
     "SCIENTIFIC", "HUMANITARIAN"
53   DATA "INTRINSIC", "FOUNDATION", "SUPPORTIVE", "LIMITED", "UNIFORM",
     "DISADVANTAGED"
54   DATA "STRUCTURE", "PERFORMANCE", "REINFORCEMENT", "GROUPING",
     "CURRICULUM", "ENVIRONMENT"
55   DATA "OBJECTIVE", "BOARD", "EXPERIMENT", "POLICY", "TEACHING",
     "EXAGGERATION"
1∅∅  PRINT W$(RND(12)-1);"   ";
11∅  PRINT W$(RND(12)+11);"   ";
12∅  PRINT W$(RND(12)+23)
13∅  PRINT
14∅  FOR Q = 1 TO 6∅∅ : NEXT Q
15∅  GOTO 1∅∅
```

It is easy to substitute words of the reader's choice by changing the DATA statements (5∅,51,52,53,54,55). The program chooses its first word from the first 12 words of the data list, the second word from words 13-24 and the third word from words 25-36 in the DATA list. You may be interested in seeing what happens if you replace one of the words between quotes with a blank space between quotes.

YOU CAN MAKE ME "TALK" PLAIN OR FANCY. THEN TRY CONVERSATION.
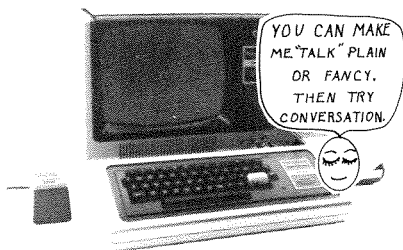
A SECOND SENTENCE GENERATOR

Let's write a program to generate more sentences. Consider the sentence:

HELEN QUICKLY SAW THE RED FOX.

The parts of speech are

NOUN   ADVERB   VERB   ARTICLE   ADJECTIVE   NOUN

Another sentence having the same pattern is:

JOHN SADLY SWEPT THE DIRTY FLOOR.

Let's write a program that will accept 10 nouns, 5 adverbs, 6 verbs and
7 adjectives. We shall start by setting aside the needed space by using
 CLEAR and   DIMension statements.

```
  2   CLEAR 1ØØØ
  5   CLS
 1Ø   DIM N$(9), A$(4), V$(5), J$(6)
 2Ø   S$ = " "
```

Next, we shall write a portion of the program to permit us to type in the
words of our choice.

```
1ØØ   PRINT "TYPE  TEN  NOUNS.  DEPRESS    (ENTER)    AFTER EACH WORD."
11Ø   FOR K = Ø TO 9
12Ø      PRINT K+1;
13Ø      INPUT N$(K)
14Ø   NEXT K
15Ø   PRINT
```

That wasn't so hard was it? The computer numbers the stored words Ø to 9,
but humans seem to prefer 1 to 1Ø, so that is what we used for each.
Let's continue.

```
2ØØ   PRINT  "PLEASE TYPE 5 ADVERBS.  DEPRESS    (ENTER)    AFTER EACH WORD."
2Ø5   PRINT "ADVERBS USUALLY END IN -LY."
21Ø   FOR K = Ø TO 4
22Ø      PRINT K + 1;
23Ø      INPUT A$(K)
24Ø   NEXT K
25Ø   PRINT
3ØØ   PRINT  "PLEASE TYPE 6 VERBS (PAST TENSE).  DEPRESS   (ENTER) AFTER
      EACH WORD."
31Ø   FOR K = Ø TO 5
32Ø      PRINT K + 1;
33Ø      INPUT V$(K)
34Ø   NEXT K
35Ø   PRINT
```

176

```
4ØØ  PRINT "PLEASE ALSO TYPE 7 ADJECTIVES.  DEPRESS  (ENTER)  AFTER
     EACH WORD."
41Ø  FOR K = Ø TO 6
42Ø     PRINT K+1
43Ø     INPUT J$(K)
44Ø  NEXT K
45Ø  PRINT
5ØØ  PRINT "THAT IS FINE .  NOW, I'LL CREATE SOME SENTENCES FOR YOU."
6ØØ  FOR Q = 1TO 1ØØØ : NEXT Q
61Ø  PRINT
62Ø  PRINT   N$(RND(1Ø)-1); S$; A$(RND(5)-1); S$; V$(RND(6)-1); S$; "THE"
     ; S$; J$(RND(7)-1); S$; N$(RND(1Ø)-1); " . "
63Ø  GOTO 6ØØ
```

RUN ‌ENTER‌

A list of the major STRING instructions follows:

Function	Operation	Examples
ASC(*string*)	Returns ASCII code of first character in string argument.	ASC(B$) ASC('H'')
CHR$(*code exp*)	Returns a one-character string defined by *code*. If *code* specifies a control function, that function is activated. (See Lesson 4 for equivalent list.)	CHR$(34) CHR$(K)
FRE(*string*)	Returns amount of memory available for string storage. Argument is a dummy variable.	FRE(A$)
INKEY$	Strobes keyboard and returns a one-character string corresponding to key pressed during strobe (null string if no key is pressed). Usually used as	INKEY$

3ØØ A$ = INKEY$: IF A$="" THEN 3ØØ ELSE PRINT A$;

NOTE: There is no space between the quote marks. Thus IF A$ is the null string, the program loops back to strobe the keyboard again. ELSE it prints the value typed and continues with the next instruction.

Function	Operation	Examples
LEN(*string*)	Returns length of *string* (zero for null string).	LEN(A$+B$) LEN("HOURS")

Function	Operation	Examples
LEFT$(*string*,*n*)	Returns first *n* characters of *string*.	LEFT$(A$,1) LEFT$(L1$+C$,8) LEFT$(A$,M+L)
MID$(*string*,*p*,*n*)	Returns substring of *string* starting at position *p* in *string* and containing the next *n* characters.	MID$(M$,5,2) MID$(M$+B$,P,L-1)
RIGHT$(*string*,*n*)	Returns last *n* characters of *string*.	RIGHT$(NA$,7) RIGHT$(AB$,M2)
STR$(*numeric exp*)	Returns a string representation of the evaluated argument. This converts numeric variables to string variables.	STR$(1.2345) STR$(A+B*2)
STRING$(*n*,"*char*")	Returns a sequence of *n* "*char*" symbols using first character of *char*. Useful in creating borders or divisions of the output.	STRING$(30,".") STRING$(25,"A") STRING$(5,C$)
VAL(*string*)	Returns a numeric value corresponding to a numeric-valued string. This is the inverse of STR$( ).	VAL("1"+A$+"."+C$) VAL(A$+B$) VAL(G1$)

(*string*   may be a string variable, expression, or constant.)

To <u>concatenate</u> strings (put them together), use the + operator as in

$$"2" + "YOU" + "76" = 2YOU76$$

Note that it is important to include desired leading and trailing spaces inside the quotes if you wish them to appear both in concatenation and in   PRINT   "YOUR NAME IS   ";N$; " ."

178

A CIPHER PROGRAM

The following program creates a simple, but reasonably secure cipher by first changing each character into its numerical ASCII equivalent, using the ASC( ) instruction, adding a computer-generated random number to the ASCII code, moving the result to the desired range $48 \le N \le 90$, and then changing the modified ASCII code into an appropriate symbol using CHR$( ). Decipherment is done using the reverse process. You may change the basic key by changing instruction 135 to R = Q*R for some other appropriate Q-value. But don't make the change unless you understand random-number generators. The choice of Q is critical and depends upon the number system used.

```
1Ø    DIM C(5ØØ)
4Ø    L = Ø
5Ø    INPUT "PLEASE TYPE YOUR KEY--POSITIVE TO ENCIPHER, NEGATIVE TO DE-
      CIPHER.";K
6Ø    R = ABS(K) : R1 = R
7Ø    PRINT "PLEASE TYPE COMPLETE MESSAGE FOLLOWED BY THE SYMBOL ↑ ."
1ØØ   P$ = INKEY$ : IF P$ = "" THEN 1ØØ ELSE PRINT P$;
1Ø5   IF P$ <= "/" THEN 1ØØ
112   IF P$ = "↑" THEN 2ØØ
115   L = L+1
117   IF K< Ø THEN R1 = -1*R1
12Ø   C(L) = ASC(P$)+R1
125   IF C(L)> 9Ø THEN C(L) = C(L) - 43
13Ø   IF C(L)< 48 THEN C(L) = C(L) + 43
135   R = 197*R
137   R = R-1ØØØØ*INT(R/1ØØØØ)
139   R1 = INT(.ØØ4*R)
15Ø   GOTO 1ØØ
2ØØ   PRINT
2Ø5   IF K < Ø THEN 3ØØ
21Ø   FOR M = Ø TO L-1 STEP 5
215       FOR N = 1 TO 5
22Ø           PRINT CHR$(C(M+N));
23Ø       NEXT N
24Ø       PRINT "   ";
25Ø   NEXT M
26Ø   END
3ØØ   FOR M = 1 TO L
31Ø       PRINT CHR$(C(M));
32Ø   NEXT M
4ØØ   END
```

Note, that instruction 1ØØ uses the INKEY instruction which strobes the keyboard once. If a key is depressed during the strobe, that value is stored in variable P$, otherwise P$ contains a null (not a blank, just nothing at all). The instruction

            IF P$ = ""   THEN 1ØØ ELSE PRINT P$

sends the computer back to 1ØØ P$ = INKEY$ if P$ is blank, otherwise it prints the value of P$ (the key just depressed) on the screen and continues to the next instruction

1Ø5 IF P$ <="/" THEN 1ØØ

which essentially refuses any symbols before / (= 47 in ASCII—see Lesson 4).

112 IF P$ = "↑" THEN 2ØØ

sends the computer to 2ØØ when the message is complete, as indicated by typing ↑

Lines 115 to 15Ø encipher or decipher the message depending upon whether the key K is positive (encipher) or negative(decipher). Each user will presumably have a different numerical key.

2Ø5 IF K < Ø THEN 3ØØ

sends the computer to 3ØØ if it is deciphering the message, and prints the entire message as one string (blanks between words were not enciphered—they could have been, but weren't).

IF K >= 0 then the computer continues with instruction 21Ø to 26Ø which displays the enciphered message in blocks of five symbols (a standard cipher practice).

ROBOT COUNSELOR

The following program, which runs on any 4K TRS-80, gives a brief insight into what programs that have more memory available can be expected to do.

```
 1  REM      ROBOT COUNSELOR
 2  RANDOM
 5  CLS
 7  CLEAR 1ØØ
1Ø  DIM G$(9)
11  G$(Ø) = "HMM...MMM...,VERY INTERESTING"
12  G$(1) = "STRANGE.  NOT ABNORMAL, YOU UNDERSTAND, JUST STRANGE"
13  G$(2) = "FASCINATING"
14  G$(3) = "UNUSUAL, BUT PERFECTLY REASONABLE UNDER THE CIRCUMSTANCES"
15  G$(4) = "THERE MAY EASILY BE MORE TO IT THAN THAT"
16  G$(5) = "THAT MAY BE AN EXAGGERATION"
17  G$(6) = "VERY COMMON IN TODAY'S WORLD"
18  G$(7) = "BEWILDERING.  I DO NOT UNDERSTAND IT"
19  G$(8) = "I HOPE TOMORROW WILL BE BETTER"
2Ø  G$(9) = "SURPRISING IN THE LIGHT OF YOUR BACKGROUND"
5Ø  PRINT  "I'D LIKE TO HELP YOU.  MAYBE TOGETHER WE CAN WORK OUT YOUR
    PROBLEMS.  PLEASE ANSWER ME WHEN I ASK YOU QUESTIONS."
```
continued....

180

```
52    FOR Q = 1 TO 400 : NEXT Q
53    PRINT
55    PRINT "JUST TYPE YOUR ANSWERS, THEN DEPRESS THE WHITE (ENTER) KEY."
57    FOR Q = 1 TO 300 : NEXT Q
59    PRINT
60    INPUT "PLEASE TYPE YOUR FIRST NAME AND DEPRESS (ENTER) KEY.";N$
61    PRINT : PRINT : PRINT  "HI "; N$
70    PRINT : PRINT
80    FOR Q = 1 TO 900 : NEXT Q
100   PRINT : PRINT  "HOW DO YOU FEEL, ";N$;"?  PLEASE ANSWER IN ONE OR TWO
      WORDS."
110   INPUT R$
115   FOR Q = 1 TO 500
120   PRINT:PRINT:PRINT "OH, I SEE.  THAT IS ";G$(RND(10)-1);"."
130   FOR Q = 1 TO 800 + RND(1000) : NEXT Q
139   PRINT
140   PRINT "WHO DO YOU THINK MAKES YOU FEEL ";R$;"?"
150   INPUT R2$
155   IF R2$ = "ME" THEN R2$ = "YOU ALONE"
156   IF R2$ = "MYSELF" THEN R2$ = "YOU YOURSELF"
157   IF R2$ = "YOU" THEN R2$ = "THE TRS-80"
160   PRINT:PRINT
163   FOR Q = 1 TO 100 + RND(500):NEXT Q
165   PRINT "WELL NOW, ";N$; ",THAT IS CERTAINLY ";  G$(RND(10)-1);"."
170   FOR Q = 1 TO 500 + RND(600):NEXT Q
180   PRINT:PRINT "DO YOU REALLY BELIEVE ";R2$;"MAKES YOU FEEL ";R$;"?"
190   INPUT A$
200   IF A$ = "NO" THEN PRINT "WELL ";N$;", PLEASE BE FRANK WITH ME.   IT
      SEEMS ";G$(RND(10)-1);":": GOTO 140
210   PRINT:PRINT:
220   FOR Q = 1 TO 10 + RND(1000):NEXT Q
225   PRINT "THAT IS ";G$(RND(10)-1);".   IT IS ";G$(RND(10)-1);"."
228   PRINT "THINK ABOUT IT A BIT."
229   PRINT:PRINT "IT IS INDEED ";G$(RND(10)-1);"." ;PRINT:PRINT
230   PRINT "REALLY NOW, TELL THE TRUTH.  PLEASE BE FRANK WITH ME."
235   PRINT "IT SEEMS ";G$(RND(10)-1);"."
240   GOTO 100
```

After you have run the program a few times, change some of the phrases
stored in G$( ) in lines 11 to 20. See if you can produce a more inter-
esting ROBOT COUNSELOR program.

LOGICAL OPERATORS

Have you wondered how the computer handles IF instructions?

The instruction

IF X >13   THEN 42∅ ELSE 3∅

examines the number stored in location X and if X is greater than 13, sends the computer to statement 42∅ for its next instruction but if X is less than or equal to 13 sends the computer to statement 3∅ for its next instruction.

The instruction

IF X >125   THEN X = 125

examines the variable X and if X is greater than 125, sets X = 125.

Actually the expression X >125 is evaluated as a logical expression.

If the current value of X makes X > 125 a TRUE statement, then the value of the logical expression

(X > 125) is −1.

If the current value of X makes X > 125 a FALSE statement, then the value of the logical expression (X > 125) is ∅.

You can examine this by running the program

```
1∅∅  Y = 25
11∅  FOR K = 1 TO 8
12∅  X = K * Y
13∅  PRINT K,X,(X > 125)
14∅  NEXT K
```

which produces the output

1	25	∅
2	5∅	∅
3	75	∅
4	1∅∅	∅
5	125	∅
6	15∅	−1
7	175	−1
8	2∅∅	−1

The actual values (TRUE = −1, FALSE = ∅) used are unimportant at this stage, and may vary on different computers. (LEVEL I TRS-80's use TRUE = 1, FALSE = ∅ .)

182

If the value produced represents TRUE (i.e., -1) then the remaining statements on the IF line are carried out (possibly several statements separated by colons).

If the value produced represents FALSE (i.e., Ø) then the computer skips the rest of the instructions in that statement line (possibly several instructions) and goes to the next line or the ELSE statement for its next instruction.

It is possible to <u>combine</u> statements using logical operators:

AND   If both expressions are true   TRUE (=-1) otherwise FALSE (=Ø)

OR   IF either expression is true   TRUE (=-1) otherwise FALSE (=Ø)

NOT   Interchanges TRUE (-1) and FALSE (Ø).

EXAMPLE:

```
1ØØ  INPUT A,B
11Ø  PRINT "A=";A, "B=";B
12Ø  IF A> B   THEN PRINT "A IS GREATER THAN B"
13Ø  IF (A > Ø) AND (A < 1Ø) THEN PRINT " Ø < A < 10." ELSE PRINT "A<= Ø or
     A>= 1Ø."
14Ø  IF (B > 2Ø) OR (B < -2Ø) THEN PRINT "ABS (B)> 2Ø."
15Ø  IF ((A> Ø) AND (B > Ø))  OR ((A < Ø) AND (B< Ø)) THEN PRINT "A*B> Ø."
     ELSE PRINT "A*B <= Ø."
16Ø  GOTO 1ØØ
```

Actually the AND, OR, NOT logical operations also may be used to test individual <u>bits</u> in the binary storage of TRS-80 words since the operations perform Boolean operations on the bits of numbers as well as on logical statements. Readers interested in such masking operations should first consult chapter 8 of the TRS-80 Level II BASIC REFERENCE MANUAL and then a text on Boolean (logical) algebra.

<u>PRACTICE SET 14</u>

1. Change the DATA words in the Buzz Phrase Generator program on page **175** to generate buzz phrases for an area with which you are familiar. Show it to a friend.

2. Change the G$( ) statements in the ROBOT COUNSELOR program pages **180-81** to phrases you like better. Improve the logic (flow) of the program as well if you can. Demonstrate the resulting program to a friend.
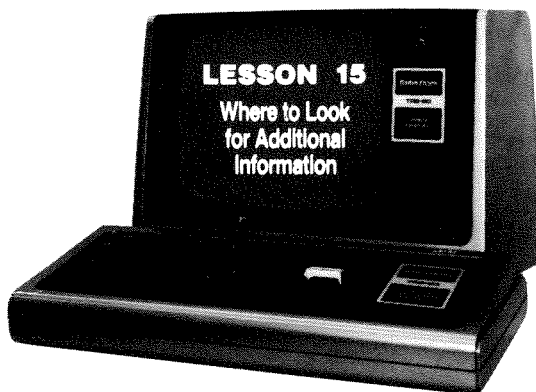
WHERE TO LOOK FOR ADDITIONAL INFORMATION

Your first source of additional information is the LEVEL I or
LEVEL II BASIC REFERENCE MANUAL that came with your TRS-80. You should
also be receiving (free) the Radio Shack Micro Computer Newletter if you
filled out the card requesting it that came with your TRS-80.

Fellow computer buffs are another excellent source of information.
If you are in real difficulty, don't hesitate to telephone or write to
The Radio Shack Computer Services Center in Ft. Worth. They maintain a
staff of knowledgeable people who seem both willing and able to help TRS-
80 owners. The address is

        Computer Services
        900 Two Tandy Center   Telephone:
        Ft. Worth, TX 76102   1-(800)-433-1679

They do not assist in debugging or writing programs, but will glad-
ly assist you if you have questions about what a given BASIC or Z-80
instruction does or about the TRS-80 hardware.

Most computer programmers, either hobbyists or professionals, learn
most about computing by burning midnight oil at the computer. One learns
to compute by computing, and then by analyzing the results.

There seem to be clubs, conventions and magazines devoted to almost
any hobby that interests you, and computers are no exception. There are
even special clubs, conventions and magazines devoted exclusively to
microcomputers. Some are so highly specialized that they concentrate on
the TRS-80 microcomputer. Ask around your own area and see what micro-

184

computer clubs and/or conventions are available. If no microcomputer club exists in your area, organize one yourself.

No list of microcomputer magazines (journals) can ever be complete—new ones seem to start up almost monthly; others drop into oblivion. Some are excellent; others are a waste of funds. Most will send you a sample copy if you ask for one, then you can judge how well it fits your particular interests.

Here are some that we take, with our favorites indicated by a star ✳. Prices were current at time of printing.

Micro Computer Conference Proceedings

The Best of the West Coast Computer Faires, 333 Swett Road, Woodside, CA, 94062. Set of 4 $53; Vols I (1976) & III (1978) $13.72 ea.; Vols II (1977) & IV (1979) $14.78 ea.

✳ Show & Tell microCOMPUTER Conference Proceedings, Department of Mathematics, University of Oklahoma, Norman, OK, 73019. (1978) $7.50; (1979) $10; (1980) $10; (1981) $10. Conference held in May each year.

Micro Computer Periodicals

✳ Byte, Byte Subscriptions, P.O. Box 590, Martinsville, NJ, 08836, monthly, $15 per year. Both hardware and programs. Some technical material.

Computronics, TRS-80 Monthly News Magazine, P.O. Box 149, New York City, NY, 10956, monthly, $24 per year. Not worth the cost in my judgment—request an examination copy and judge for yourself. Magazines change.

The Computing Teacher, Computing Center, Eastern Oregon State College, La Grande, OR, 97850, bi-monthly, $8 per year.

✳✳ Creative Computing, P.O. Box 789-M, Morristown, NJ, 07960, monthly, $15 per year. Probably the best buy available. Something for everyone. Well written and authoritative articles.

Data General News, Data General Corporation, Southboro, MA, 01772 (free—devoted to their computers).

Dr. Dobbs Journal of Computer Calisthenics & Orthodontia, The Peoples Computer Co., Box E, Menlo Park, CA, 94025, monthly, $12 per year. Advanced and excellent.

✳ Games, Games Productions, Inc., 515 Madison Ave., New York City, NY, 10022, bi-monthly, $5.97 per year. Not computer oriented, but lots of good ideas you can use.

Games & Puzzles, Edu-Games (U.K.) Ltd., P.O. Box 4, London, England, N64DF

✳ *Journal of Recreational Mathematics*, Baywood Publishing, 120 Marine, Farmingdale, NY, 11735. Interesting problems, some computer related. Quarterly, $25. ($10 to individuals, home address, personal check.)

Personal Computing, Circulation, Personal Computing, 1050 Commonwealth Ave., Boston, MA, 02215, monthly, $14 per year.

✳ *Computer Music Journal*, ISSN 0148-9276, MIT Press Journals, 28 Carleton St., Cambridge, MA, 02142 Price $5 per issue, $20 per year.

✳ *PROG 80*, Softside Publications, 17 Briar Cliff Dr., Milford, NH, 02055, monthly, $15 per year. Lots of goodies for the serious TRS-80 programmer.

Radio Shack Microcomputer Newsletter, Radio Shack, One Tandy Center, Fort Worth, TX, 76102. TRS-80 owners receive free for 6 months, then $12/year.

Recreational Computing, People's Computer Company, 1263 El Camino Real, Box E, Menlo Park, CA, 04025, 6 issues per year, $10.00.

The Recreational Programmer, Computer Software, P.O. Box 2571, Kalamazoo, MI, 49003, bi-monthly, $12 per year.

Softside, Softside Publications, 17 Briar Cliff Dr., Milford, NH, 02055, monthly, $15 per year. Lots of BASIC games in this one.

T-PAL, The Mail Mart, Box 11102, San Francisco, CA, 94101, monthly, $24 per year. Seems expensive for what you get. Ask for a sample copy, it should have improved.

✳ *TRS-80 Computing*, Computer Information Exchange, P.O. Box 158, San Luis Rey, CA, 92068, monthly, $15 per year. Lots of goodies here.

✳ *80-US*, 80-NW Publishing, 3220 N. 32nd St., Tacoma, WA, 98407, sample copy, $3, $16 per year. Six issues annually.

PAGE, Bulletin of the Computer Arts Society, John Lansdown, 50151 Russell Square, London, WCIB4JX, England, quarterly, $51 per year (students, half-price). Or write Kurt Lauckner, Math. Dept., Eastern Michigan University, Ypsilanti, MI, 48917. Devoted to computer assisted art.

TRS-80 Users Group, 7554 Southgate Rd., Fayetteville, NC, 28304, monthly, $12 per year. An informal publication for amateurs that is well worth the price.

✳ *80 Microcomputing*, Subscription Dept., PO Box 981 Farmingdale, NY 11737. Started in January 1980 and well worth the $25/year. This monthly slick magazine devoted to TRS-80 does not pull punches and keeps you informed on the latest updates, changes and evaluations of hardware and software related to the TRS-80.

Popular Computing, PO Box 307, Martinsville, NJ, 08836

Where to Buy Programs

We don't purchase many programs; we write our own because it is more
fun. Most of the microcomputer magazines and conference proceedings carry
programs. Some, you can obtain on tape at a modest cost. Most carry ads
for programs that will run on your TRS-80. Some sell programs that are
definite "rip-offs". Others are bargains. Few have adequate documenta-
tion with them. Still, you will almost certainly be tempted to buy some,
just to try them out. Read the ads. Go to your local Radio Shack (or
Radio Shack Computer Center, if you have one...many middle-to-large cities
do). Go to any other computer stores in your area and ask to see the pro-
grams in action. (Don't let your local computer store disparage the TRS-
80. Some will because they do not sell them.) Attend a microcomputer
club in your area. Ask about available software.

If you haven't seen Leo Christopherson's Android Nim (with sound),
you should. Nim isn't much of a game, but the animation is superb (1980
price, $14.95, runs on 16K Level II TRS-80). If you wish a TRS-80 chess
or checker playing program, look over the results from the most recent
national microcomputer chess or checkers tournament. There are a dozen
programs available for under $20 each, and the quality varies considerab-
ly. Read the comments and the ads in whatever microcomputer periodicals
you and/or your libaray take.

You should probably write to:

                    People's Software
                    Computer Information Exchange
                    Box 158
                    San Luis Rey, CA 92066

They put a couple of dozen programs on a tape and sell it for $10.95
(see Lesson 7). They also sell more expensive programs, like People's
Pascal ($ 23.50). New programs are added frequently. Drop them a card
asking for a current price list.

HAVE FUN! GROW WITH YOUR NEW HOBBY. READ, EXPERIMENT, DISCUSS YOUR
EXPERIENCES.   HAVE FUN!

LESSON 16
Additional Basic

Your TRS-80 Level II BASIC Reference Manual contains details of the available Level II BASIC instructions. If you have an expansion interface, line printer, or disk you will find additional instructions defined in the accompanying reference manuals. Here are several additional Level II BASIC instructions you may find convenient.

Additional Keyboard Commands	Examples

AUTO *mm*,*nn*

AUTO 1ØØ   [ENTER]

Turns on automatic line numbering beginning with *mm* and in steps of *nn*. If ,*nn* is omitted, the default step size is 1Ø.

AUTO 1ØØ,5   [ENTER]

AUTO 3ØØ, 20   [ENTER]

DELETE *mm*-*nn*

DELETE 41Ø   [ENTER]

Deletes program steps from line *mm* to line *nn* inclusive. Both *mm* and *nn* should be actual statement numbers in the program.

DELETE 8Ø-23Ø   [ENTER]

LIST *mm*-*nn*

LIST 27Ø-4ØØ   [ENTER]

Lists program from line *mm* to line *nn*.

LIST 14Ø-   [ENTER]

LIST -27Ø   [ENTER]

LIST   [ENTER]

RUN *mm*

RUN   [ENTER]

Executes program beginning at line *mm*.

RUN 45Ø   [ENTER]

SYSTEM

SYSTEM   [ENTER]

Enter monitor mode for loading Z-80 language program from cassette tape. Frequently used with advanced commercial programming.

Additional Program Instructions	Examples

TRON

TRON

Turns on TRACE program in which variable values are shown on screen to help you debug program. TRACE program is always in the TRS-80, as part of your Level II BASIC ROM.

TROFF

TROFF

Turns TRACE program off. The TRACE program is still in the computer.

USR(n)

USR($\emptyset$)

Branches to a Z-80 machine language subroutine already entered by the user. You'll need to use POKE first.

RESTORE

RESTORE

Resets the DATA pointer to the first DATA statement to permit program to reuse the same data.

GOSUB nn

GOSUB 46$\emptyset$

Branch to the subroutine beginning at line number nn.

GOSUB 37$\emptyset\emptyset$

RETURN

RETURN

Each subroutine must contain at least one RETURN statement. This sends the program back to the line following the GOSUB that branched to the subroutine.

ON α GOTO mm, nn, pp, qq, ' ' '

ON Z GOTO 2$\emptyset\emptyset$, 3$\emptyset\emptyset$, 4$\emptyset\emptyset$

If the value of INT(α), where α is a variable or expression, is one of the numbers 1,2,3,4,' ' ',k then go to the statement number in that position on the list of statement numbers mm, nn, pp, qq,' ' '.

ON K+2 GOTO 7$\emptyset\emptyset$,7$\emptyset\emptyset$,4$\emptyset\emptyset$,3$\emptyset\emptyset$,5$\emptyset\emptyset$

ON α GOSUB mm, nn, pp, qq, ' ' '

ON K GOSUB 33$\emptyset\emptyset$, 2$\emptyset\emptyset\emptyset$, 4$\emptyset\emptyset$

Same as ON α GOTO except the RETURN pointer is set so program will return to statement following ON α GOSUB.... when RETURN is encountered.

Arithmetic Functions

Function	Operation(unless noted otherwise, −1.7E+38<=exp<=1.7E+38)	Examples
ABS(exp)	Returns absolute value.	ABS(L*.7) ABS(SIN(X))
ATN(exp)	Returns arctangent in radians.	ATN(2.7) ATN(A*3)
CDBL(exp)	Returns double-precision representative of exp.	CDBL(A) CDBL(A+1/3#)
CINT(exp)	Returns largest integer not greater than exp. Limits: −32768<=exp<+32768.	CINT(A#+B)
COS(exp)	Returns the cosine of exp;assumes exp is in radians.	COS(2*A) COS(A/57.29578)
CSNG(exp)	Returns single-precision representation, with 5/4 rounding in least significant decimal when exp is double-precision.	CSNG(A#) CSNG(.33*B#)
EXP(exp)	Returns the natural exponential, e^{exp}=EXP(exp).	EXP(34.5) EXP(A*B*C−1)
FIX(exp)	Returns the integer equivalent to truncated exp (fractional part of exp is chopped off).	FIX(A−B)
INT(exp)	Returns largest integer not greater than exp.	INT(A+B*C)
LOG(exp)	Returns natural logarithm (base e) of exp. Limits: exp must be positive.	LOG(12.33) LOG(A+B+B)
SGN(exp)	Returns −1 for negative exp; 0 for zero exp; +1 for positive exp.	SGN(A*B+3) SGN(COS(X))
SIN(exp)	Returns the sine of exp; assumes exp is in radians.	SIN(A/B) SIN(90/57.29578)
SQR(exp)	Returns square root of exp. Limits: exp must be non-negative.	SQR(A*A − B*B)
TAN(exp)	Returns the tangent of exp; assumes exp is in radians.	TAN(X) TAN(X*.01745329)

exp may be any numeric variable, constant or computed expression.

There are a number of other Level II BASIC instructions available for special use on your TRS-80. If your programming is advanced enough to need these extra instructions see the <u>TRS-80 Level II BASIC Reference Manual</u> and the reference manuals accompanying any additional equipment you may have (Disk, Lineprinter) etc.

Three examples are:

POKE *location, value*
Loads *value* into memory *location*. The arguments must be decimal numeric variables, constants or functions with $0 \leq \text{value} \leq 225$. Used in advanced programming to change memory content or display screen and with USR( )function. See Chapter 8 of Level II BASIC Reference Manual.

POKE 16396,23
will disable the BREAK key. This is sometimes used to prevent unsophisticated users from BREAK -ing into a program.

PEEK *(address)*
Returns the (decimal) value stored in the memory *address* specified. Also, makes values of Z-80 machine language variables available to BASIC programs. See Chapter 8 of <u>Level II BASIC Reference Manual</u>.

```
10   FOR K = 0 TO 12288
20   PRINT K;PEEK(K),
30   NEXT K
```

will display the location and contents of your Level II BASIC ROM. It won't mean much to you, but it will do it.

RANDOM
is frequently executed <u>once</u> near the beginning of a program to reseed the RND( ) instruction. Do <u>not</u> use RANDOM inside of a loop or where it will be used repeatedly in your program.

```
1   REM      program name
2   RANDOM
```

LESSON 17
More Micro-Research
Problems

EXAMPLE 17-1

Let us examine a program designed to determine the factors of a given integer. Forms of this program have appeared in several magazines, books and even in tape form. The given program works. It produces the desired factors for any positive integer Z. However, it is easy to improve upon the program, as we shall see.

First examine the original program to see why it works.

```
1Ø   REM   FIRST WORKING ATTEMPT TO FACTOR A NUMBER
1ØØ  INPUT "TYPE NUMBER TO BE FACTORED"; Z
11Ø  PRINT  "THE PRIME FACTORS OF";Z;"ARE";
12Ø  N = Z
13Ø  F = 2
14Ø  IF N/F < > INT (N/F) THEN 2ØØ
15Ø  PRINT F;
16Ø  N = N/F
17Ø  IF N = 1 THEN STOP ELSE 14Ø
2ØØ  F = F + 1
21Ø  IF F < = Z THEN 14Ø
```

The heart of the program is instruction 14Ø

        IF N/F < > INT(N/F) THEN 2ØØ

which uses the integer function INT( ) to determine whether or not N/F is an integer (i.e., whether or not F is a factor of N). If F is not a factor of N, the program jumps to instruction 2ØØ which increases F by 1. If new F < = Z the program loops back to instruction 14Ø to determine whether or not the new F is a factor of N. If F is a factor of N, the value of F is printed, N is replaced by N/F and if the new N = N/F <> 1, the test is repeated. When either of the conditions N = 1 or F > Z occurs, the original number Z is completely factored.

192

The program also uses an axiom many experienced programmers have found worthwhile

Note that the program inputs the value of Z but then sets N = Z and works with N. We could have ignored this motto here, but many experienced programmers will tell you they stick to it in their own work. It avoids trouble.

The program produces the prime factors of any positive integer of six or fewer digits you put in. Try it and see. Then examine the program to see exactly how it works.

If you have any trouble following the program, make a chart of variables in the program and step the program through. If your INPUT value is Z = 12, your chart will contain

Z	N	F	PRINTED OUT
12			The prime factors of 12 are
	12	2	2
	6	2	2
	3	3	3
	1 STOP		

After you understand how the program works, you may wish to create a flow-chart of the logic involved. Flowcharts are frequently used to express the logic involved in more complicated problems. This particular program is probably not involved enough to require a flowchart, but it is easy to "see" the flowchart of a simple problem.
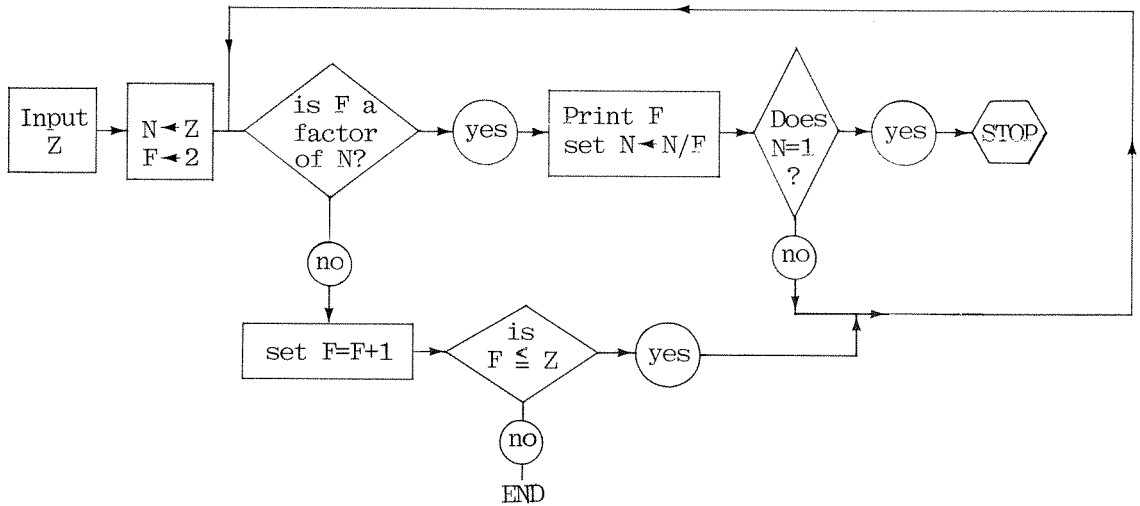
Your flow chart should look <u>something</u> like this:
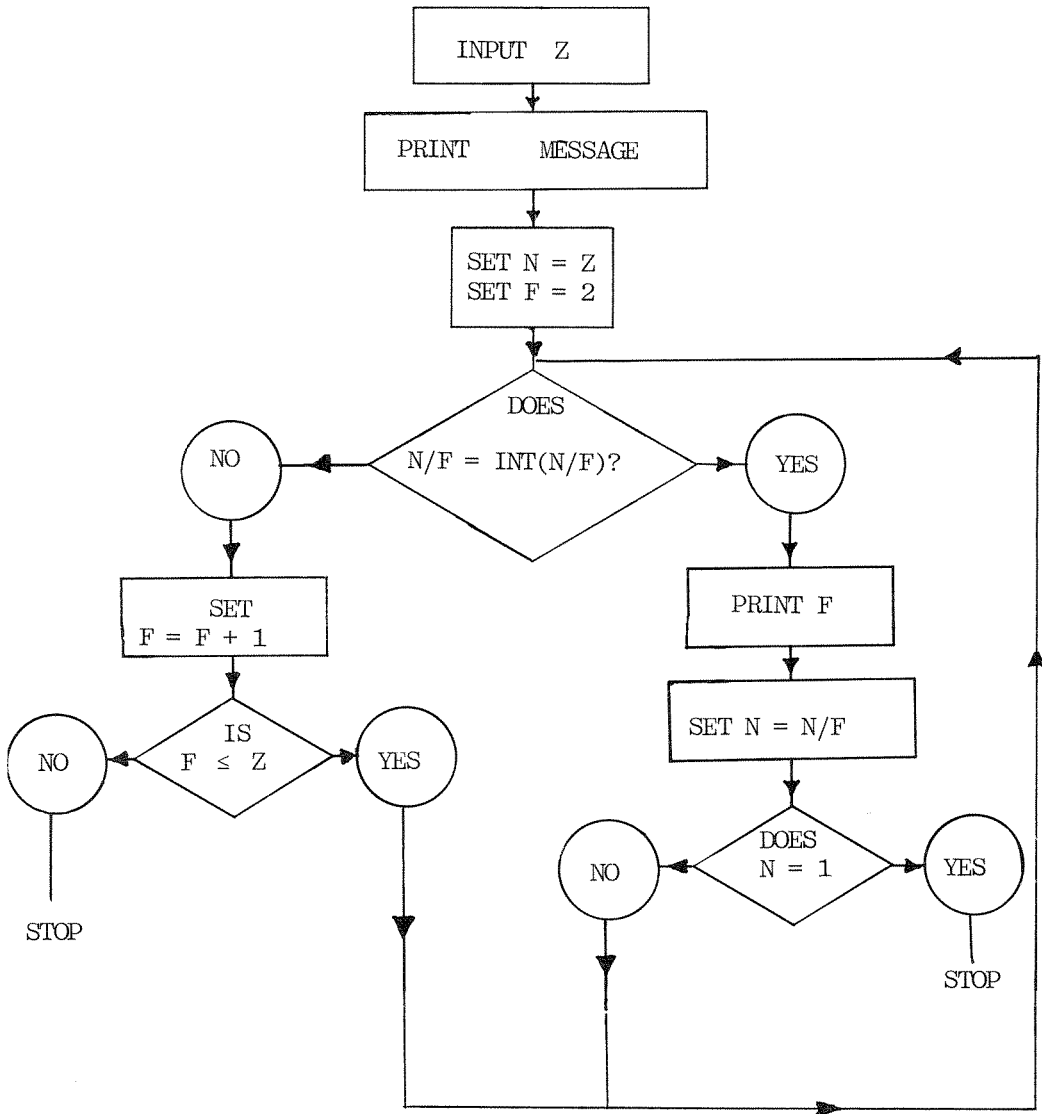
Note how easy it is to explain what the program is doing using a flow chart.

Actually, although the program in question has been published in several versions, it is an example of deplorably poor programming practice. It works fairly well on small values, but note the difference between Z = 1847 and Z = 1848 in the time required. Try it yourself before continuing.

Furthermore, Z = 1950 takes :01 second to factor and Z = 1951 takes 1:01 seconds to factor. This seems a long time for a powerful computer. Of course, it does do a lot of arithmetic in that time. In fact, that is just the trouble -- it does too much arithmetic. The program tests F = 2, 3, 4, 5, 6, 7, 8, 9,..., Z repeatedly. However, if we factor out all the 2's at the beginning, then there is no reason to also try 4, 6, 8, 10, etc. This should enable us to almost cut the time by half.

This can be done by making the following changes:

<table>
<tr><td>OLD PROGRAM</td><td>FIRST REVISION</td></tr>
</table>

OLD PROGRAM	FIRST REVISION
1∅ REM FIRST WORKING ATTEMPT TO FACTOR A NUMBER	1∅ REM FIRST WORKING ATTEMPT TO FACTOR A NUMBER
1∅∅ INPUT "TYPE NUMBER TO BE FACTORED " ; Z	1∅∅ INPUT "TYPE NUMBER TO BE FACTORED " ; Z
11∅ PRINT "THE PRIME FACTORS OF "; Z; "ARE ";	11∅ PRINT "THE PRIME FACTORS OF "; Z; "ARE ";
12∅ N = Z	12∅ N = Z
	► 125 IF N/2 <> INT(N/2) THEN 135
13∅ F = 2	► 13∅ PRINT " 2";
	► 132 N = N/2
	► 134 GOTO 125
	► 135 F = 3
14∅ IF N/F <> INT(N/F) THEN 2∅∅	14∅ IF N/F <> INT(N/F) THEN 2∅∅
15∅ PRINT F;	15∅ PRINT F;
16∅ N = N/F	16∅ N = N/F
17∅ IF N = 1 THEN STOP ELSE 14∅	17∅ IF N = 1 THEN STOP ELSE 14∅
2∅∅ F = F + 1	► 2∅∅ F = F + 2
21∅ IF F < = Z THEN 14∅	► 21∅ IF F < = N THEN 14∅

In instruction

    21∅ IF F < = Z THEN 14∅

there is really no reason to try F values that are larger than the current value of N (instead of the original Z), so we replace instruction 21∅ with

    21∅ IF F < = N THEN 14∅

This should cut our time even further. Make the above changes in 125,

13Ø, 132, 134, 135, 2ØØ, and 21Ø. Then, try the program again.

N	Old Program Time	New Program Time
12	:01	:005
1847	:58	:30
1848	:01	:01
12345	:26	:14
–6	:01 *ignores sign*	:005
8.2	Runs through 21Ø and stops.	

Even the above program would not satisfy most professional program-
mers. The biggest complaints would probably be that the program mal-
functions if negative values are used, and that the data was not tested
to be sure the input value was an integer.

The problem of fractional values is readily overcome by inserting:

1Ø5 IF Z <> INT(Z) THEN PRINT "PLEASE USE INTEGER VALUES" : GOTO 1ØØ

Negative values could also be handled as improper input, but since
they can be factored, we prefer another solution.

115 PRINT   "(" ; SGN(Z); ")" ,
12Ø N = ABS(Z)

This will produce either –1 or 1 in parentheses as a first factor.
We include the parentheses since mathematically, neither –1, nor 1, is a
prime. By ending instruction 115 with a comma, instead of a semicolon,
we also provided extra space between the unit and the prime factors.

As long as we are improving our program, we might as well make full
use of the TRS-80's arithmetic ability by requesting it to use DOUBLE
PRECISION arithmetic, which permits us to work with numbers as large as
16 digits, instead of 6. We can do this by adding:

10 DEFDBL Z, N, F

However, if we do this, we will be using the much slower double pre-
cision arithmetic, even when our number to be factored has 6 or fewer
digits. To avoid this, insert:
     1Ø   DEFDBL Z
    1Ø6   IF Z > 999999 THEN DEFDBL N, F

Many programmers would prefer to print a blank line or two and loop
the program back to the beginning rather than stopping it once the number
is factored. This is easily done by changing instruction 17Ø to

     17Ø   IF N=1 THEN 9Ø ELSE 14Ø

and adding:

196

```
 9Ø PRINT : PRINT
3ØØ GOTO 9Ø
```

Our revised program now reads:
```
 1Ø DEFDBL Z
 9Ø PRINT : PRINT
1ØØ INPUT    "TYPE NUMBER TO BE FACTORED "; Z
1Ø5 IF Z <> INT(Z) THEN PRINT  "PLEASE USE INTEGER VALUES" : GOTO 1ØØ
1Ø6 IF Z > 999999 THEN DEFDBL N, F
11Ø PRINT    " THE PRIME FACTORS OF "; Z ; "ARE " ;
115 PRINT    "(" ; SGN(Z); ")",
12Ø N = ABS(Z)
125 IF N/2 <> INT(N/2) THEN 135
13Ø PRINT "2";
132 N = N/2
134 GOTO 125
135 F = 3
14Ø IF N/F <> INT(N/F) THEN 2ØØ
15Ø PRINT F;
16Ø N = N/F
17Ø IF N = 1 THEN 9Ø ELSE 14Ø
2ØØ F = F + 2
21Ø IF F < = N THEN 14Ø
3ØØ GOTO 9Ø
```

Although our new program takes more lines to write than the original program, actually it produces results in less than half the time, and works for many additional input values.

There are still three important cases to be considered. What happens if Z = Ø? Try it and see. If the output does not satisfy you, change the program further by adding:

IF Z = Ø THEN PRINT " *some appropriate message.*"

Mathematically oriented readers may prefer to make the program even faster by using only <u>prime</u> values for F instead of 2, 3, 5, 7, 9, 11, 13, 15, ...Z. This will indeed save considerable time on factoring larger numbers, but requires that a list of prime numbers be stored or generated. We leave this as an exercise for those desiring additional speed. The critical statements are

```
READ    F
DATA    2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47,
DATA    53, 59, 61, 67, ..., etc.
DATA
```

where the DATA statements contain a list of all primes that are $\leq \sqrt{999999999999999}$ or < 100000000   if 16-digit arithmetic is being used.

The point of this exercise is <u>not</u> to illustrate how to write a fact-
oring program, but rather to show you that it is very easy to <u>improve</u> most
programs, including commerical programs written in books (even this one).
Just because a program "works" does not mean it is a good, or even an
acceptable, program. The critical questions are:

1. Does it do the job for which it was designed? (Does it work?)
   If not, it is useless.

2. Can I improve it noticeably <u>without much effort</u>? If so, improve it
   and test the new program.

3. If it requires much effort to improve the program, will the results
   be worth the effort? (If the program will seldom be used again, and
   if it does not take an unreasonable amount of computer time as it
   is, it probably is <u>not</u> worth the effort to improve it <u>unless</u> you
   want to use it to brag a bit.)

4. You should always ask yourself if the method (algorithm) used by
   the original programmer is the best one you can think of.

Experienced programmers have a saying that contains much truth.

> Only after you have written, debugged, and tested
> a program, do you understand it well enough to
> program it. If it is a major program, after you
> get the program all up and running smoothly, scrap
> the entire program and THINK carefully about it.
> Then start over from scratch and write a better
> program.

MORE microRESEARCH PROBLEMS

We examine another example in detail before trying new micro-Research Problems. We consider the four-digit case first, then after that is investigated, ask you to expand it in micro-research Problem 18, as suggested in the STEPS FOR COMPUTER ASSISTED PROBLEM SOLVING.

EXAMPLE 17-2

Let ABCD be a 4-digit number such as 4721

```
1ØØ LET X = A
    A = ABS |A-B|
    B = ABS |B-C|
    C = ABS |C-D|
    D = ABS |D-X|
    PRINT A ; B ; C ; D
    IF A+B+C+D <> Ø THEN 1ØØ ELSE STOP
```

Your first problem is to write a BASIC program that will show that each 4-digit starting-value number will converge to ØØØØ in 9 or fewer steps.

Write your own program before continuing.

EXAMPLE

$ABCD = 4721$
$X = 4$
$A = |4-7| = 3$
$B = |7-2| = 5$
$C = |2-1| = 1$
$D = |1-4| = 3$

$NEW\ ABCD = 3513$
$X = 3$
$A = |3-5| = 2$
$B = |5-1| = 4$
$C = |1-3| = 2$
$D = |3-3| = Ø$

$NEW\ ABCD = 242Ø$
$X = 2$
$A = |2-4| = 2$
$B = |4-2| = 2$
$C = |2-Ø| = 2$
$D = |Ø-2| = 2$

$NEW\ ABCD = 2222$
$X = 2$
$A = |2-2| = Ø$
$B = |2-2| = Ø$
$C = |2-2| = Ø$
$D = |2-2| = Ø$
$NEW\ ABCD = ØØØØ$
$STOP$

There are several ways this could be done. One of the more obvious is:

```
21Ø FOR M = 1 TO 9
22Ø    FOR H = Ø TO 9
225       PRINT   "WORKING ON M, H = "; M; H
23Ø          FOR T = Ø TO 9
24Ø             FOR U = Ø TO 9
25Ø                A = M
26Ø                B = H
27Ø                C = T
28Ø                D = U
29Ø                PRINT   "STARTING WITH A, B, C, D = " A; B; C; D

3ØØ                N = Ø
31Ø                X = A
32Ø                N = N+1
33Ø                A = ABS(A-B)
34Ø                B = ABS(B-C)
35Ø                C = ABS(C-D)
36Ø                D = ABS(D-X)
37Ø                PRINT   A; B; C; D
38Ø                IF A + B + C + D > Ø THEN 31Ø
39Ø                PRINT M;H;T;U; "TAKES ";N; "STEPS TO CONVERGE TO ØØØØ."

4ØØ             NEXT U
41Ø          NEXT T
42Ø       NEXT H
43Ø NEXT M

5ØØ PRINT
51Ø PRINT   "FINISHED WITH 4-DIGIT TESTS."
```

This program runs through all of the four-digit starting-values, printing out new values as they are developed, and printing out the number N of steps each starting-value requires to converge to ØØØØ before going to a new starting-value whenever ABCD = ØØØØ.

Put it on your computer and run it. It is interesting to watch, but it sure takes a <u>long time</u> to test 9000 cases!

Of course, you don't really <u>need</u> to watch, since, if a given starting value ever fails to produce ØØØØ, the program will continue trying, and the change in pattern will be clear.

However, if we are <u>not</u> going to watch the program, it can be speeded up a lot by eliminating statements 29Ø and 37Ø. Try it and see. Be sure you also recognize how to tell if this revised program ever finds a starting value which does not converge to ØØØØ.

It is still a long program, and besides, it doesn't really do what we set out to do (to show that every 4-digit starting-value converges to

$\emptyset\emptyset\emptyset\emptyset$ in <u>nine</u> or fewer steps).

Hmmm...let's store the largest N-value in another variable called L. (Why not in B for biggest?) We can do that by inserting:

```
2Ø5 L = Ø
385 IF L > = N THEN 4ØØ
395 L = N
515 PRINT    "LARGEST N VALUE IS " ; L
```

Let's see how that works.

```
2Ø5 L = Ø
21Ø FOR M = 1 TO 9
22Ø    FOR H = Ø TO 9
225       PRINT  "WORKING ON M, H =   "; M; H
23Ø       FOR T = Ø TO 9
24Ø          FOR U = Ø TO 9
25Ø             A = M
26Ø             B = H
27Ø             C = T
28Ø             D = U

3ØØ             N = Ø
31Ø             X = A
32Ø             N = N+1
33Ø             A = ABS(A-B)
34Ø             B = ABS(B-C)
35Ø             C = ABS(C-D)
36Ø             D = ABS(D-X)
37Ø             PRINT A;B;C;D;
38Ø             IF A + B + C + D > Ø     THEN 31Ø
385             IF L > = N THEN 4ØØ
39Ø             PRINT M;H;T;U; "TAKES ";N; "STEPS TO CONVERGE TO ØØØØ."
395             L = N

4ØØ             NEXT U
41Ø          NEXT T
42Ø       NEXT H
43Ø NEXT M

5ØØ PRINT
51Ø PRINT  "FINISHED WITH 4-DIGIT TESTS."
515 PRINT  "LARGEST N VALUE IS " ; L
```

When we ran the above program, we discovered L = 8. You can modify the program to determine all 4-digit starting-values that require 8 steps to converge to $\emptyset\emptyset\emptyset\emptyset$.

(continued from Lesson 5)

18. a. Extend Example 17-2 to 5-digit starting-values A B C D E

$$A = X$$
$$A = ABS(A-B)$$
$$B = ABS(B-C)$$
$$C = ABS(C-D)$$
$$D = ABS(D-E)$$
$$E = ABS(E-X)$$

etc. Let's print out each set (like we did in 29∅ and 37∅) to see what is going on at the early stages of our investigation.

It may come as a shock to you that not all values converge to ∅∅∅∅∅. Indeed, apparently quite a few converge to ∅∅∅XX (where the digits XX are alike, but not necessarily ∅∅), and then enter a long loop which eventually returns to ∅∅∅XX.

b. Modify the program or write a new one to investigate what really does happen to 5-digit starting-values.

c. Extend your investigations to include 6, 7, and 8-digit starting-values and 2 and 3-digit starting-values, as well as the 4 and 5-digit values already investigated.

Would you like to conjecture that all starting-values converge to ∅∅...XX, but that we can guarantee XX = ∅∅ only if the number of digits in the starting-value is a power of 2? Why, or why not?

19. Write a program that will shuffle 52 cards in the computer and deal out 4 bridge hands, and then bid one of the hands, with human input for bids on the other three hands.

20. Write a program to create Haiku poetry.

21. Write a program that will accept seven subscripted variables V(0), V(1),...,V(6) and will test to determine whether or not any of the variables contain equal values. If all seven variables are different, print "ALL VARIABLES ARE DISTINCT." Otherwise, print the subscripts and the common value of those variables containing the same value. If there is more than one set of subscripted variables with the same values, the program should also indicate the subscripts and common values of other sets.

$$V(0) = 4$$
$$V(1) = -3$$
$$V(2) = 1.765$$
$$V(3) = 4$$
$$V(4) = -2$$
$$V(5) = -3$$
$$V(6) = 4$$

The output should be

```
SUBSCRIPTS = 0 3 6   VALUE = 4
SUBSCRIPTS = 1 5     VALUE = -3
```

22. Write a program to accept one of the three symbols from the key-board $-$ $\emptyset$ $+$. The program should then examine a string of DATA (from DATA statements or from tape) and if

   $-$ was depressed at the keyboard, display the number and sum of the negative DATA values.

   $+$ was depressed at the keyboard, display the number and sum of the positive DATA values.

   $\emptyset$ was depressed at the keyboard, display the number of zeros in the DATA.

23. A theorem from number theory states that every integer N is expressible in the form

   $$N = X^2 + Y^2 - Z^2 \quad \text{where X, Y, and Z are integers.}$$

   a. Write a program that will accept values of N between -100 and $+100$ and print out N (as the sum of two integer squares minus another integer square) before returning to accept another value N .

   b. Enclose your program in a FOR...NEXT loop to test all $-100 < N < 100$.

   c. Can you speed your program up so it could feasibly work for $-5000 < N < 5000$?

24. Euclid's proof that there exist infinitely many prime integers uses the expression

   $$N_k = 1 + (2*3*5*7*11*...*P_k)$$

where the product in the parentheses is the product of the first k positive primes.

Write a computer program to determine the first 20 values of N_k.

Extend your program to determine which of these N_k values are prime and what the factors of the non-prime N_k are. (Note: not all N_k are prime, but all of N_k's prime factors are larger than P_k and less than $(1 + \sqrt{N_k})$.

25. Knowing that our calendar repeats on a 400-year cycle, determine on which day of the week the 13th day of the month is most apt to occur. (Is it really Friday?)

26. Write a program to make a table showing the values of

$$N \qquad N^2 \qquad R$$

(Where R is the remainder when N^2 is divided by 12.)

for $N = 1, 2, 3, \ldots, 20$. Then jump to a conclusion (call it a conjecture, if anyone asks you) and then either prove or disprove your conjecture.

27. Determine the sum of the first k terms in the series

$$1 + 22 + 333 + 4444 + \ldots$$

For a solution, consult page 807 (E 1405), American Mathematical Monthly Vol 67 (1960).

28. Determine all of the 4-digit integers which are perfect squares and in which all four of the digits are even. Your final program should produce the results in less than 3 seconds of TRS-80 Level II BASIC RUN time. If it takes much more than that, please do some additional analysis and try again.

29. Devise a program that will draw a dart target on the TRS-80 screen (squares are much easier to draw than circles), with numbers shown in each area (including negative numbers outside the target if you wish). Then permit a player to toss 5 darts by typing coordinates X,Y of a point. Add a little "wobble" to the point chosen by using X = X + (RND(9) – 5): Y = Y + (RND(5) –3) or some such device before display-ing SET(X,Y) as a (first blinking, then permanent) pixel on the screen. Your program should also keep a running total of the number of points the player made.

After the program is up and running--rewrite it so that the cen-ter of the target is not at the center of the screen. Also, permit the gamemaster to assign the values to each target area before the game starts.

30. The reverse of an integer (whole number) is the number obtained by writing its digits in reverse order--that is, the reverse of N=1089 is R=9801 and the reverse of N=13 is R=31.

a) Your project is to find values of N such that N does not equal its reverse, N≠R, but the product N*R is a perfect square. Since $1089*9801 = 10673289 = (3267)^2$ such numbers clearly exist. How many can you find?

31. The numbers 12 and 21 are reverses of one another, as are 144 and 441, since they contain the same digits, but in reverse order. Interestingly, $12^2 = 144$ and $21^2 = 441$. Find other pairs N and N´ such that N and N´ are reverses with N≠N´, and such that N^2 and $(N´)^2$ are also reverses.

32. You borrow an amount P dollars at I percent per annum interest rate on the unpaid balance. You agree to pay off M dollars per month. Write a program that will print out a table showing at the end of each month:

Amount of interest paid that month	Amount of principal paid that month	Remaining principal	Total Interest paid to date	Total paid to date

Note that if in the last month (principal + interest) < M you should not have to pay the full $M.

33. Lesson 4 contained a program designed to help you learn where the various pixel blocks were located on the screen. Extend the idea by first having the player type a difficulty number N where 1 is hard and 9 is easy, perhaps using

INPUT "PLEASE TYPE DIFFICULTY NUMBER BETWEEN 1 (HARD) AND 9 (EASY); N

Then give the player 1+RND(N) chances to locate the lighted block with blinking pixel, before clearing the screen and choosing another randomly positioned lighted block.

34. Write a program to accept a positive integer K and then type out K primes of the form $N^2 + 1$.
Example: If K = 3
the primes are 2, 17, 37
each of which is prime and of the form $(N^2 + 1)$.

35. Write a program containing 20 or fewer BASIC instructions without using a PRINT" (PRINT *followed by quotes*) instruction to make a caricature of a face or person or animal.

36. Design a program which will serve as a timer for a series of 10-minute speeches. When the time keeper types RUN [ENTER] your program should display 10 MINUTES LEFT on the screen in normal size letters. The number of minutes should change every minute. When the speaker gets to 1 minute left the program should display some attention getting device on the screen and the switch to double size letters and display

     0 MINUTES 59 SECONDS

             58 SECONDS

    etc. until 0 SECONDS is reached, whereupon the screen should go wild (possibly including an audio blast if you are equipped for audio output).

37. Prove or disprove that, for N a positive integer,

    $$N^4 + 2N^3 + 2N^2 + 1 \text{ is}$$

    never a perfect square. (Don't expect the computer to do this for you.)

38. Your local Utopia Society decides to replace their weekly Bingo session with a Numbers Betting Game to be available in gas stations and grocery stores. People can place bets on any three digit number from 000 to 999. The winning number is the three digits in the thousands places of the number of shares sold on the New York Stock Exchange that day. The payoff is 500 times the amount bet for bets up to $20. For bets above $20 the payoff is $10,000 plus 100 times the amount bet.

    Your first task is to write a program to simulate the betting system and come up with estimates of likely profits or losses. You should also discover (with or without using a computer)what amount of bet will produce the greatest payoff for the bettor. If all bettors wagered this amount (best possible for them, worst for the Utopians), how much would you expect the Utopia Society to make or lose on each approximately $10,000 that was bet? Assume that 10¢ of each dollar bet goes to the person taking the bet, and 15¢ of each dollar bet is other overhead.

39. Compute and print the number 2^{75} exactly.

40. Find all positive integers X,Y,Z such that

$$X^2 + Y^2 = Z^2 \text{ with } Z < 1000.$$

Compare the speed of your program with the speed of programs written by your friends.

41. a) Two rods of equal length are divided into 250 and 243 equal parts respectively. Their ends are coincident. Find the divisions which are nearest together.

b) Same problem but divide the two equal rods into M and N equal parts. Write a program to accept positive integer input M and N and display the two values which are nearest together.

42. Consider the recursive relation defined by:

Let   B = The biggest number that can be made by rearranging the digits of N_k.

S = The smallest number that can be made by rearranging the digits of N_k.

N_{k+1} = B – S

a) If N_0 is a four-digit starting value, one of two things happen:

i) If all the digits of N_0 are identical, the $N_{k+1} = 0$ for all K ≥ 0.

ii) If N_0 has at least two distinct digits, then the relation converges to $N_{k+1} = 6174$ which then repeats forever. Investigate this phenomenon.

b) If N_0 is a six-digit integer, there are 384 non-zero possible values of N_1, some of which lead to the repeater 631764, but some of which do not. Investigate other starting values N_0 having 6 digits.

c) Investigate the N_{k+1} = BIG – SMALL phenomenon for starting values of other sizes.

43. It is well-known that the reciprocals of integers form repeating decimals of period p (we say, p = 0 if the expansion terminates as on 1/2 = .500; p = 1 for 1/3 = .33; etc.) Our problem is to create a table which will give the smallest positive integer N(K) such that its reciprocal has a repeating decimal expansion of length K.

44. If N_0 is a positive integer, it can be shown that the recursive relation N_{k+1} = (The sum of the squares of the digits of N_k) will either converge to 1 or will reduce to a self-repeating cycle 37, 58, 89, 145, 42, 20, 4, 16, 37, ...

   a) Do so. Can you prove that every positive starting value N_0 will eventually either converge to 1,1,1, or to the cycle 20, 4, 16, 37, ...

   b) The related problem for the sum of other powers of the digits also merits investigation. Do so.

45. It is a common practice in mapping to approximate the area of a lake or island or other irregular shape by using an irregular polygon, whose area is easier to compute than is the area inside a curved boundary.

   Write a program into which the number N of vertices and the X and Y coordinates of the vertices (X_i, Y_i) of the polygon in consecutive order may be input and the program will then compute the polygonal area. It will be necessary to either find or devise a suitable formula. Remember this polygon is not a regular polygon since its sides may be of unequal length.

46. Given two values A and B, determine values X, Y, and Z such that

$$\frac{A}{X} = \frac{X}{Y} = \frac{Y}{Z} = \frac{Z}{B}$$

   What will your program do if A and B have different signs?

47. Liouville discovered an interesting procedure for producing sets of positive integers with the property that the sum of their cubes is equal to the square of their sum. One such set is

$1^3 + 2^3 + 3^3 + 4^3 = (1 + 2 + 3 + 4)^2$. A bit of induction will show that for all N, $1^3 + 2^3 + 3^3 + ...+ N^3 = (1+2+3+...+N)^2$. However, it is not known whether or not Liouville's method will produce all such sets. Write a program to produce such sets in a given range. Investigate the problem.

48. Write a program that will compute how much an investment of P dollars will be worth Y years from now if it is invested at I percent per annum interest, compounded N times per year. A useful formula is:

$V = P*(1 + I/(100*N))^{N*Y}$ (Where V is the value after Y years.)

49. Design a program to coach emergency first-aid treatment at the "Boy Scout" level. The program should ask questions answerable by YES or NO and prescribe action based on the response.

Is patient bleeding severely?

No   Yes

Bind wounds to stop flow of blood. Do not use a tourniquet unless life is threatened.

Is patient conscious?

No   Yes

Is there any evidence that patient has taken poison or excessive medications?

No   Yes

Type number to indicate likely material ingested:

1   Prescription medicine
2   Other medicine
3   Soaps, detergents, cleansers
4   Insecticides, bugkillers
5   Caustics: lye, Drano, bowl cleaner, Liquid Plumber
6   Gasoline-like products
7   Paints, stains, enamels
8   Heavy oils or grease

ON P GOTO ...

(1) (2) (3) (4) (5) (6) (7) (8)

50. Either find a ten-digit positive integer N with each digit different, and such that N is also a perfect 10th power of an integer, or show that no such N exists.

51. Consider a large lattice of points   (100 x 100 would be large in this case and a big sheet of graph paper will do nicely) with equal spacing in the x and y directions. Our real problem is to place as many points as possible on the lattice subject to the restriction that no subset of three points shall form a right triangle. Before tackling the real problem, consider the much simpler problem of a program to find and print out 6 or 8 patterns of points on the lattice which have the property that no three points form a right triangle and that the set is maximum in the sense that if one more point were added to the given arrangement, a right triangle would be formed. Possibly placing points down the diagonal has this property. Then try to answer (with proofs) the questions:

   a) What is the largest number of points which may be so placed?

   b) What is the smallest number of points in a maximum set?

   c) Restate the problem so that using every point of two adjacent sides of the square excepting the corner point is not a solution and solve the new problem.

52. Write a program to sort a set of 25 names alphabetically. Ponder this one. Sorting represents one of the most time-consuming uses of modern computers. Consider sorting and entering the approximately 100,000 grades received by 20,000 students each term at a major university.

53. J. Maxfield has shown (Math Magazine, Vol. 43, #2 March, 1970, pp.64-67) that given a sequence of digits $K = d_1 d_2 d_3 \ldots d_n$, there exists an integer N such that N! begins with the sequence of digits K. Write a program to create a table such that the smallest such positive integer N(K) is determined for each K $\leq$ 100. You may also wish to print out the floating point value of N! if it will be available at no additional cost. Eventually, we would like a formula for N(K) = smallest positive integer N such that N! begins with the sequence of digits K, but that may be too much to hope for. Your table might well resemble the one below:

K	N(K)	N!
1	1	1
2	2	2
3	9	362880
4	8	40320
5	7	5040
6	3	6
7	6	720
.	.	.
.	.	.
.	.	.

54. Mathematicians have proved that there exist arbitrarily long strings of consecutive positive integers such that each of the consecutive integers has as a factor, a perfect square greater than one (probably different squares for different integers). This may be restated as follows:

> Given a positive integer N, there exist strings of N consecutive positive integers each of which contains a factor which is a perfect square greater than one.

If N = 3   48, 49, 50 contain 4^2, 7^2, 5^2 as factors.
             98, 99, 100 contain 7^2, 3^2, 10^2 as factors.

If N = 4   242, 243, 244, 245 contain 11^2, 3^2, 2^2, 7^2 factors.

If N = 7   217070, 217071, 217072, 217073, 217074, 217075, 217076
contain as
factors 7^2 3^2 2^2 113^2 11^2 5^2 4^2

Your problem is to find the first (i.e., involving smallest integers) string of N consecutive integers each of which contains a square >1 as a factor for N = 2, 3, ..., 10.

55. This one is tough! Mathematicians have shown a similar theorem to that given in Problem 54 holds with "perfect square" replaced by "perfect cube" or even "perfect Kth power". Investigate this for K = 3, 4, 5, and N = 2, 3, ..., 10. (Or even N = 2, 3, 4, 5, 6, for starters.)

Examples: 3rd Powers (K = 3)   4th Powers (K = 4)

    N=2   N=2
            8∅   81   8∅   81
             8   27   16   81

    N=3   N=3
         1375 1376 1377   33614 33615 33616
          125   8   27   24∅1   81   16

    N=4
      22624 22625 22626 22627
          8   125   27 1331

56. Find a set of distinct positive integers, each of which is less than 100, such that it satisfies both of the following conditions.

a) No combination (or subset) of them added together will total 100.

b) The set contains as many distinct, positive integers as possible.

Hint: You may be able to solve this without using a computer.

57. The square of the number 36363636364 has the unusual property (in base 10) that the second half of the square is an exact duplicate of the digits in the first half

$$(36363636364)^2 = 13223140496,13223140496$$

as shown by the central comma above.

Such periodic squares also exist in bases other than 10. For example, in base 4,
$$(21213_4)^2 = 11301,11301_4 = 378225_{10} = (615_{10})^2$$

Your problem is to write a program that will examine perfect squares, expressed in some base $B > 1$, (use base $B = 10$, if you wish) and find perfect squares whose expression in base B is periodic (period 2 or more). See <u>Mathematics Magazine</u>, March 1975, p. 97.

58. An integer (whole number) greater than 1 is said to be <u>prime</u> if its only positive factors are itself and one. There are many 4 x 4 arrays of digits such that each row and each column is a 4-digit prime.

Example:

				Where		
6	1	8	9		6189	6359
3	Ø	2	3		3Ø23	1Ø21
					5273	8273
5	2	7	3		9137	9337 are each prime.
9	1	3	7			

a) Find some additional such arrays. Then, hunt for arrays whose main (upper left to lower right) diagonal also contains a prime.

b) What happens if you try to find 5 x 5 arrays containing 5-digit primes in each row, each column and main diagonal?

59. Write a program to plot a graph $Y = f(X)$.

Your program may request the user to put the function in a certain statement. It should ask the user for what range of X-values he wishes the graph plotted and then compute (or ask the user) for the largest and smallest Y-value that will be used. It should then scale the function suitably for display on the CRT screen. Your program should also display for the user (perhaps using PRINT @ 960, "   ") the X-range and the Y-range covered by the graph.

60. Find the smallest positive integer M such that

$$N = 7*10^M + 1 \qquad \text{is \underline{not prime}.}$$

212

61. Find and print all the five-digit positive integers which contain exactly the same digits as their trebles. The leading digit must not be zero.

> Example: $10035 * 3 = 30105$

Please also record the computer time used on your final run. Compare with that of other TRS-80 users you know.

62. Many people from Archbishop John Tillotson (c. 1650) through Kurt Vonnegut (1950) and beyond have toyed with the idea that if enough monkeys were permitted to pound typewriters for long enough, all the great classics of literature would eventually be reproduced. We can easily improve the chances of something legible being produced by picking keys in the approximate order of their frequency in normal English rather than at random. Write a program that will print out the following symbols, with the given approximate frequency per 1000. To simplify the problem we restrict our selection of punctuation to "space", and have omitted V, K, Q, X, J, and Z, which have very low frequency in English.

KEY	space	E	T	A	O	N	I	R	S	H
FREQUENCY/1000	223	100	77	62	62	54	54	54	46	38

KEY	D	L	F	C	M	U	G	Y	P	B
FREQUENCY/1000	31	23	23	23	23	15	15	15	15	15

You may be able to further improve the likelihood of obtaining at least occasional words by including digraphs (two-letter groups) as well as single letters. English digraph frequencies are approximately

DIGRAPH	TH	HE	ER	AN	IN	ON	RE	AT	ED	ST	ND	ES
FREQUENCY/1000	38	31	18	17	17	16	14	14	13	12	12	12

Additional frequency data for English, French, Spanish, German, and Italian are available in Sophisticated Ciphers, available from Mu Alpha Theta Math Club, Room 423, 601 Elm street, Norman, OK, 73019, or at your library.

63. Find all n-digit positive integers in which the n-digit number is an exact n-th power of an integer.

> Examples: $81 = 9^2$ $125 = 5^3$

64. Two positive integers are amicable (friendly) if the proper (including one, but not including the number itself) divisors of each add up to the other. 220 has divisors 1, 2, 4, 5, 10, 11, 20, 22, 44, 55, 110, whose sum is 284. 284 has divisors 1, 2, 4, 71, 142, whose sum is 220, thus 220 and 284 are said to be amicable. Some other amicable pairs are (17296, 18416) and (1184, 1210). Write a program that will find all amicable pairs between two given bounds S and B and which will also keep track of any number which has a divisor sum greater than B, so these can be used in a future run with (new S) = (old B) and a larger B. Your final program should also account for any "carry over" values from previous runs.

65. Amicable pairs (problem 64) have been generalized to amicable chains (sociable chains) in which the sum of the proper divisors of each number in the chain is the next number and the last member of the chain has the first member as the sum of its divisors. The only chains generally known are the two four-link chains (found in 1965):

    (2115324, 3317740, 3649556, 2797612) and
    (1264460, 1547860, 1727636, 1305184),

    the five-link chain:

    (12496, 14288, 15472, 14536, 14264),

    and a stupendous 28-link chain which contains the number 14316. However, there are others.

    a) Write a program to investigate amicable chains containing 50 or fewer links.

    b) What happens to chains that do not loop back to the starting value?

66. Write a program that will match up 50 men and 50 women as dance partners. Your first attempt can use numbers, if you wish, but the final result should be two lists; one with the men's names in alphabetical order and showing the randomly matched partner's name, the other with the womens' names in alphabetical order and showing the name of each woman's partner (hopefully, the same matching as above). Each person should be matched with one and only one partner and that partner should be of the opposite sex.

67. a) Write a computer program to accept three values M, D, Y and determine whether or not they are acceptable values for Month, Day, Year, in that order.

    b) Extend the above program to accept two sets of three numbers and if both are acceptable as Month, Day, Year values, determine the number of days between them, including both dates in your count. Example: 2, 20, 1978 to 2, 22, 1978 is 3 days.

68. A computer is apt to obtain different values for
    $F(N) = 1*2*3*...(N-1)*N$ and for $G(N) = N*(N-1)*...3*2*1$.
    Furthermore, the difference is apt to be non-trivial. For $N = 30$,
    the difference can be greater than a quintillion (and that's more
    than the number of cents in the national debt). Investigate this
    phenomenon, using a program of your own devising, or using:

```
1ØØ FOR N = 1Ø TO 5Ø
11Ø     F = 1
12Ø     G = N
13Ø     FOR K = 2 TO N
14Ø         F = F*K
15Ø         G = G*(N-K+1)
16Ø     NEXT K
17Ø     PRINT N, F; G, ABS(F-G)
18Ø NEXT N
```

69. Mathematicians have shown that given a sequence of digits
    $S = d_1 d_2 d_3 ... d_n$ there exists an integer N such that 2^N begins with
    the sequence S. (As a matter of fact, for 2 we may substitute any
    positive integer which is not a power of 1Ø, i.e., not $1Ø^0=1$, $1Ø^1=1Ø$,
    $1Ø^2=1ØØ$, etc.)

    Your problem is to produce a table giving S, N, 2^N for S = 1 to 1ØØ
    such that N(S) is the <u>smallest</u> positive integer such that $2^{N(S)}$
    begins with S. I think the first few values are:

S	Smallest N>Ø	2^N
1	4	16
2	1	2
3	5	32
4	2	4
5	9	512
6	6	64
7	46	7Ø368744177664
8	3	8
9	53	etc.
1Ø	1Ø	
11	5Ø	

70. Problem 11 (Lesson 5) asks you to generate and print out <u>in increas-
    ing order</u> of size, the positive integers of the form $N = 2^k \cdot 3^m$
    where $k \geq Ø$ and $m \geq Ø$. Generalize this to integers of the form
    $A^k \cdot B^m$ where A and B are read using an INPUT statement.
    Note that A and B need not be prime. Your program should work if
    A = 6 and B = 10.

215

71. Use an array containing the proper fractions in lowest terms to establish a "position number" for each entry. In the array each entry in row k shall have (k+1) as denominator and the numerators in a given row shall be in increasing order.

1/2

1/3   2/3

1/4   3/4

1/5   2/5   3/5   4/5

1/6   5/6

1/7   2/7   3/7   4/7   5/7   6/7

1/8   3/8   5/8   7/8

1/9   etc.

1/1∅   etc.

.
.
.

Since only those proper fractions in lowest terms are included, the rows are not of easily predictable length.
Establish a position number $N(\alpha)$ for each fraction α in the array by simply counting the entries, row after row, from left to right until α is reached.

Thus
$N(1/2) = 1$
$N(1/3) = 2$
$N(1/4) = 4$
$N(2/3) = 3$
$N(3/4) = 5$

and so forth. In this fashion one establishes that

$N(1/13) = 46$
$N(15/16) = 79$
$N(1/100) = 3004$
$N(3/500) = 75917$
$N(7/1240) = 467048$ (or do they?)

The challenge is to determine formulae or an algorithm that will enable you to do either (or both) of the following

1. Given a proper fraction α in lowest terms, determine $N(\alpha)$.
2. Given an integer K, determine the corresponding proper fraction α such that $K = N(\alpha)$. Use a computer to help if you can.

72. Program a game of your own choice on the computer.

73. The problem of placing eight queens on an 8x8 chess board so that no queen can attack any other queen (queens attack any piece they can reach by moving in a straight line horizontally, vertically, or on either diagonal) is an old one. You may have considered the problem before. You will find it discussed on p. 161-2 of W. W. Rouse Ball's interesting book, _Mathematical Recreations & Essays_ or in Chapter 10 of M. Kraitchik's admirable book, _Mathematical Recreations_, both of which should be available at your local library.

You may write a program to solve the problem if you wish, but the problem proposed here is to examine the following program and see if you can figure out how it works and why it produces all 92 possible solutions to the eight queens program.

```
   2  CLS
   4  PRINT "    EIGHT QUEENS PROBLEM    (RUNNING TIME ABOUT 4Ø MINUTES)"
   6  PRINT
   8  PRINT "         1    5    8    6    3    7    2    4   MEANS"
  1Ø  PRINT " THE QUEEN IN THE FIRST COLUMN IS ON ROW 1;"
  12  PRINT "   THE QUEEN IN THE SECOND COLUMN IS ON ROW 5;"
  14  PRINT "     THE QUEEN IN THE THIRD COLUMN IS ON ROW 8;ETC."
  16  PRINT
  2Ø  DIM R(8)
  5Ø  N = Ø
 1ØØ  C=Ø
 2ØØ  C=C+1
 3ØØ  R(C)=Ø
 4ØØ  IF R(C) < 8   THEN 6ØØ
 5ØØ  C=C -1
 51Ø  IF C = Ø   THEN PRINT"END OF SEARCH":STOP
 52Ø  GOTO 4ØØ
 6ØØ  R(C) = R(C) + 1
 6Ø5  IF C < 2 THEN 66Ø
 61Ø  FOR  Cl = 1 TO C -1
 62Ø     IF R(C)=R(Cl) OR C+R(C)=Cl+R(Cl) OR R(C)-C=R(Cl)-Cl THEN 4ØØ
 65Ø  NEXT Cl
 66Ø  IF C < 8 THEN 2ØØ
 69Ø  N=N+1:PRINT"SOLUTION#";N;"=",
 7ØØ  FOR Cl=1 TO 8
 71Ø     PRINT R(Cl);"  ";
 72Ø  NEXT Cl
 73Ø  PRINT
 74Ø  GOTO 5ØØ
```

74. If you are seriously interested in more advanced computing, investigate one of the following languages for your TRS-80.

"C" or "tinyC"
"PASCAL"
"Z-80" chip language (enter from BASIC II through USR( ) function)
PILOT (a language designed for Computer Assisted Instruction)

75. Study a program written by someone else, and then improve it.

A FINAL WORD from your authors

Good Programming Practices

There is no such thing as the correct programming style. Entire books have been written on the subject, but even the most skillful practitioners of the programming art disagree on what is best. However, there are several common sense principles that merit consideration. Even these are not immutable rules, but it is prudent to follow the suggestions unless you have a good reason for not doing so on a given program. Here goes:

1. Your program should, in general, read from top to bottom in a logical fashion, except for subroutines which usually are listed after the completed program. Block the various fragments into different 100's in statement numbers. Don't feel compelled to use every tenth statement number.

2. Use occasional REM statements to explain what is happening. If you must use a GOTO statement in the heart of your program, include a REM statement explaining why.
    Your programs will have a neater appearance, and be easier to debug if you place five blank spaces between REM and the start of your remark. Try it, you'll like it.

3. Whenever you use a FOR-NEXT loop, indent the instructions between FOR and NEXT three spaces. If you use nested FOR-NEXT loops, stair-step them. It makes programs much easier to understand.

4. Use the faster forms whenever feasible.
                    (See Lesson 11.)

5. Get to know your computer. Become thoroughly familiar with the quirks and excentricities of your computer and of the BASIC (or other) language you are using.

Above all:
    Don't let these or any other rules spoil your fun--but do remember that well-written, easy-to-follow programs are no harder to think or write than sloppy kluges and are much more profitable in use and maintenance. Think of a computer program as a living, growing, ever-changing thing, not as a finished, static object.

H A V E   F U N

Josephine Andree

Richard V. Andree

218

Assignment Statements

	Mod I Level I	Mod I Level II	Mod I TRSDOS	M II TRSDOS	Mod III TAPE	Mod III TRSDOS	8K COLOR	POCKET	Description
CLEAR								✓	CLEARs all data variables.
CLEAR N		✓	✓	✓	✓	✓	✓		Reserves N bytes string memory, reset variables.
DEG								✓	Converts to decimal notation.
DIM		✓	✓	✓	✓	✓	*		Dimension one or more arrays. * 1 dimension only.
DMS								✓	Converts to sexagesimal notation.
ERASE				✓					Deletes an array.
LET	✓	✓	✓	✓	✓	✓	*	✓	Optional. *DO NOT use with Color Computer.
MID$ =			✓	✓		✓			Replace old portion of string with new portion.
SWAP				✓					Exchanges the values of two variables.

Other Commands, Functions and Operations

	Level I	Level II	TRSDOS	TRSDOS	TAPE	TRSDOS	8K COLOR	POCKET	Description
ELSE		✓	✓	✓	✓	✓	✓		Secondary action clause in IF-THEN statement.
IF	✓	✓	✓	✓	✓	✓	✓	✓	Tests conditional expression in IF-THEN-ELSE.
THEN	✓	✓	✓	✓	✓	✓	✓	✓	Primary action to be taken in IF-THEN statement.
AUDIO							✓		Connects cassette audio to television speaker.
DEBUG						✓			Direct program execution under the DEBUG mode.
MERGE			✓	✓		✓			Merges ASCII disk program with resident program.
MOTOR						✓			Turns cassette motor on or off.
POKE		✓	✓		✓	✓	✓		Puts a value into a RAM memory location.
RANDOM		✓	✓	✓	✓	✓			Reseeds random number generator.
REM	✓	✓	✓	✓	✓	✓	✓		REMark; instructs computer to ignore rest of line.
SKIPF						✓			Positions cassette tape at end of next file.
DATE$				✓					Gets current date as 18 character string.
ERL		✓	✓	✓	✓	✓			Returns the line number in which an error occurred.
ERR		✓	✓	✓	✓	✓			Returns a value related to most recent error.
FRE(STR)		✓	✓	✓	✓	✓			Returns amount of unused string space.
FRE(X)		✓	✓	✓	✓	✓			Finds amount of free memory space.
INP		✓	✓	✓	✓	✓			Gets a value from specified port.
MEM	✓	✓	✓	✓	✓	✓	✓	✓	Finds the amount of free memory.
PEEK		✓	✓	✓	✓	✓			Gets value in specified memory location.
POS		✓	✓	✓	✓				Returns column position of cursor.
ROW									Gets row number where cursor is positioned.
TIME$			✓	✓	✓	✓			Returns time (24 hour format) as a string.
USR			✓	✓	✓	✓	✓		Calls a machine language subroutine.
USRn			✓	✓	✓	✓			Calls one of ten machine language routines.
VARPTR		✓	✓	✓	✓				Gets address where variable contents are stored.
EOF			✓	✓		✓	✓		End-of-file detector.
LOC			✓	✓		✓			Determines current record number of disk file.
LOF				✓		✓			Determines highest numbered record in disk file.
VARPTR(#b)				✓					Returns address of data buffer b.
AND	*	✓	✓	✓	✓	✓	✓		* USE (*) IN LI BASIC
CONCATE- NATION		✓	✓	✓	✓	✓	✓		JOIN STRINGS
EQV				✓					0 IF 1ST 1 AND 2ND 0
EXPONEN- TIATION		✓	✓	✓	✓	✓		✓	RAISE TO POWER
IMP				✓					OPPOSITE OF XOR
MOD				✓					MODULUS CALCULATIONS
NOT		✓	✓	✓	✓	✓	✓		NEGATE
OR	*	✓	✓	✓	✓	✓	✓		* USE (+) IN LI BASIC
XOR				✓					EXCLUSIVE OR

Headers (diagonal): Level I | Level II | TRSDOS | TRSDOS | TAPE | TRSDOS | 8K COLOR | POCKET

Definition Commands

Command	Level I	Level II	TRSDOS	TRSDOS	TAPE	TRSDOS	8K COLOR	POCKET	Description
DEF FN			✓	✓		✓			Defines a user-created function.
DEFDBL		✓	✓	✓	✓	✓			Define variables as double precision type.
DEFINT		✓	✓	✓	✓	✓			Defines variables as integer type.
DEFSNG		✓	✓	✓	✓	✓			Defines variables as single precision type.
DEFSTR		✓	✓	✓	✓	✓			Defines variables as string type.
DEFUSR			✓	✓	✓	✓			Defines entry point for mach. lang. subroutines.
DEGREE								✓	Set angle mode to degree.
GRAD								✓	Set angle mode to gradians.
RADIAN								✓	Set angle mode to radians.

Math Functions

Command	Level I	Level II	TRSDOS	TRSDOS	TAPE	TRSDOS	8K COLOR	POCKET	Description
ABS	✓	✓	✓	✓	✓	✓	✓	✓	Computes absolute value of X.
ACS								✓	Computes Arc cosine.
ATN		✓	✓	✓	✓	✓		✓	Computes arctangent; value returned in radians.
COS		✓	✓	✓	✓	✓		✓	Computes cosine; angle must be in radians.
EXP		✓	✓	✓	✓	✓		✓	Computes the natural antilog.
FIX		✓	✓	✓	✓	✓			Truncates all digits to right of decimal point.
HEX$		✓	✓	✓	✓	✓			Computes hexadecimal value and returns in string.
INT	✓	✓	✓	✓	✓	✓	✓	✓	Largest integer not greater than X.
LN								✓	Natural logarithm in Pocket Computer.
LOG		✓	✓	✓	✓	✓		*	Natural Logarithm *Common Logarithm in PC.
OCT$				✓					Computes octal value and returns it as a string.
RND	✓	✓	✓	✓	✓	✓	✓		RND(N) — Pseudorandom number between 1 and N.
RND(Ø)	✓	✓	✓	✓	✓	✓	✓		Generates a pseudorandom number between 0 and 1.
SGN		✓	✓	✓	✓	✓	✓	✓	Returns sign component of a number.
SIN		✓	✓	✓	✓	✓	✓	✓	Computes sine; angle must be in radians.
SQR		✓	✓	✓	✓	✓		*	Computes single-precision square root. *Keyboard
TAN		✓	✓	✓	✓	✓		✓	Computes tangent; angle must be in radians.
VAL		✓	✓	✓	✓	✓	✓		Evaluates a string as a number.

String Commands and Functions

Command	Level I	Level II	TRSDOS	TRSDOS	TAPE	TRSDOS	8K COLOR	POCKET	Description
ASC		✓	✓	✓	✓	✓	✓		Returns ASCII code of first character in string.
CDBL		✓	✓	✓	✓	✓			Converts to double precision.
CHR$		✓	✓	✓	✓	✓	✓		Returns character for ASCII or other code.
CINT		✓	✓	✓	✓	✓			Returns largest integer not greater than n.
CSNG		✓	✓	✓	✓	✓			Converts to single precision.
CVD			✓	✓		✓			Converts to double-precision after GET.
CVI			✓	✓		✓			Converts to integer after GET.
CVS			✓	✓		✓			Converts to single-precision after GET.
INKEY$		✓	✓	✓	✓	✓	✓		Gets keyboard character if available.
INSTR		✓	✓	✓					Returns starting position of substring.
LEFT$		✓	✓	✓	✓	✓	✓		Returns left portion of string.
LEN		✓	✓	✓	✓	✓	✓		Returns the number of characters in a string.
MID$		✓	✓	✓	✓	✓	✓		Returns a substring of a string.
MKD$			✓	✓		✓			Makes double-precision number a string.
MKI$			✓	✓		✓			Makes Integer a string.
MKS$			✓	✓		✓			Makes single-precision number a string.
RIGHT$		✓	✓	✓	✓	✓	✓		Returns right portion of string.
SPACE$			✓	✓					Returns a string of n spaces.
STR$		✓	✓	✓	✓	✓	✓		Converts a numeric expression to a string.
STRING$		✓	✓	✓	✓	✓			Returns a string of n identical characters.

Input/Output Commands

Command	Level I	Level II	TRSDOS	TRSDOS	TAPE	TRSDOS	8K COLOR	POCKET	Description
AREAD								✓	Contents of display read into variable.
AT	✓								Print beginning at specified screen location.
BEEP							✓		Sound buzzer n times.
CLOAD	✓	✓	✓		✓	✓	✓	✓	Loads BASIC program file from cassette.
CLOAD?		✓	✓		✓	✓		✓	Compares program on tape with resident program.
CLOADM							✓		Loads a machine language program from cassette.
CLOSE			✓	✓		✓	✓		CLOSE all open file-buffers or specified buffer b.
CSAVE	✓	✓	✓		✓	✓	✓	✓	Stores resident program on tape.
DATA	✓	✓	✓	✓	✓	✓	✓		Stores data to be accessed by a READ statement.
FIELD			✓	✓		✓			Organizes a random file buffer into fields.
GET b,r			✓	✓		✓			Gets specified record from random disk file.
INPUT	✓	✓	✓	✓	✓	✓	✓	✓	INPUTs data from the keyboard.
INPUT # − b		✓	✓		✓	✓	✓		Inputs data from specified cassette unit.
INPUT #b			✓	✓		✓			Inputs data from disk file (sequential access).
INPUT" PROMPT"	✓	✓	✓	✓	✓	✓	✓	✓	PRINTs the "PROMPT" and INPUTs data from keyboard.
INPUT#	✓							✓	Get data from tape and place in memory.
INPUT$(N)			✓	✓					Inputs n characters from the keyboard.
INPUT$(N,b)			✓	✓					Inputs n characters from sequential disk file.
LINE INPUT			✓	✓		✓			Line inputs from keyboard; <ENTER> ends input.
LINE INPUT #b			✓	✓		✓			Line inputs from disk into buffer.
LLIST	*	✓	✓	✓	✓	✓	✓		List program to line printer. *MIII Level I only.
LPRINT	*	✓	✓	✓	✓	✓			Prints items to line printer. *Yes in MIII Lev I.
LPRINT TAB		✓	✓	✓	✓	✓			Moves printer carriage to specified position.
LPRINT USING	✓	✓	✓	✓	✓	✓			Prints formatted data to line printer.
LSET			✓	✓		✓			Left-justifies data into a random access field.
OPEN			✓	✓		✓	✓		Opens file; assigns mode and buffer.
OUT		✓	✓		✓	✓			Sends value to specified port.
PAUSE								✓	Hold display 0.85 seconds and continue.
PRINT	✓	✓	✓	✓	✓	✓	✓	✓	PRINTs an item or list at current cursor position.
PRINT # − b		✓	✓		✓	✓	*		Writes data to cassette. *# − 2 Outputs to printer.
PRINT @	✓	✓	✓		✓	✓	✓		Begin printing at specified screen position.
PRINT AT	✓								PRINT AT specified screen location in Level I.
PRINT TAB	✓	✓	✓	✓	✓	✓	✓		Moves cursor right to specified TAB position.
PRINT USING		✓	✓	✓	✓	✓		✓	Formats strings and numbers for display.
PRINT#	✓							✓	Contents of data memory is recorded on tape.
PRINT# USING			✓	✓		✓			Formatted sequential write to disk.
PRINT#b			✓	✓		✓			Writes data to sequential file-buffer.
PUT			✓	✓		✓			Moves data from file-buffer to random disk-file.
READ	✓	✓	✓	✓	✓	✓	✓		READs a value from a DATA statement.
RESTORE	✓	✓	✓	✓	✓	✓	✓		Resets pointer to first item of first data line.
RSET			✓	✓		✓			Right-justifies data into random access field.
SOUND							✓		Sounds the frequency and duration specified.
SPC				✓					Prints a line of n blanks.
TAB	✓	✓	✓	✓	✓	✓	✓		Moves cursor to specified TAB position.
USING	✓	✓	✓	✓	✓	✓		✓	Defines format for PRINT and LPRINT.

Graphic Commands

Command	Level I	Level II	TRSDOS	TRSDOS	TAPE	TRSDOS	8K COLOR	POCKET	Description
CLS		✓	✓	✓	✓	✓	✓	✓	Clears the display.
CLS(C)							✓		Clears display and sets background color.
JOYSTK							✓		Returns value related to joystick positions.
POINT		✓	✓	✓		✓	✓	*	Test graphic block. *Return color if SET.
RESET	✓	✓	✓		✓	✓	✓		Turn off specified graphics block.
SET	✓	✓	✓		✓	✓	✓	*	Turn on graphic block. *SET(x,y,z) − − COLOR.

System Commands

	Mod I		MII	Mod III			
Level I	Level II	TRSDOS	TRSDOS	TAPE	TRSDOS	8K COLOR	POCKET	Description
---	---	---	---	---	---	---	---	---

Command	Level I	Level II	TRSDOS	TRSDOS	TAPE	TRSDOS	8K COLOR	POCKET	Description
AUTO		√	√	√	√	√			Numbers lines automatically.
CHAIN						√			Load and execute specified program.
CMD"A"				√					Return to TRSDOS, printing message.
CMD"C"				√		√			Compress program by removing spaces and REMs.
CMD"D"			√						Loads and executes DEBUG.
CMD"D:d"						√			Display disk directory in BASIC.
CMD"E"						√			Display most recent disk error.
CMD"I"			√			√			Returns to TRSDOS, execute specified command.
CMD"J"						√			Performs Julian calendar calculations.
CMD"L"						√			Load specified disk file from BASIC.
CMD"O"						√			Sort specified number of strings in an array.
CMD"P"						√			Returns printer status as a string.
CMD"R"			√			√			MOD I: Start clock MOD III: Display clock.
CMD"S"			√			√			Returns control to TRSDOS
CMD"T"			√			√			MOD I: Stops clock/Mod III: Turn off clock display.
CMD"X"						√			BASIC cross-reference facility.
CMD"Z"						√			Toggle "DUAL" routing.
CONT	√	√	√	√	√	√	√	√	Continues execution of program after BREAK/STOP.
DELETE		√	√	√	√	√			Erases program lines from memory.
EDIT			√	√	√	√			Puts computer into edit mode for specified line.
KILL			√	√		√			Deletes a disk file.
LIST	√	√	√	√	√	√	√	√	LIST program lines to the Video Display.
LIST ####	√	√	√	√	√	√	√	√	LIST from program line ####
LOAD			√	√					Loads program file from disk.
NAME			*			√			Renumbers resident program. *BASICR only.
NEW	√	√	√	√	√	√	√	√	Erase program from memory; initialize variables.
RENUM				√					Renumbers resident BASIC program.
RUN	√	√	√	√	√	√	√	√	Execute resident program.
RUN "FILESPEC"			√	√		√			Loads and executes specified disk program.
RUN ####	√	√	√	√	√	√	√	√	RUN from line specified by ####.
SAVE			√	√		√			Saves BASIC program on disk.
SYSTEM		√	√		√	√			Puts computer in monitor mode.
TROFF		√	√		√	√			Turns off the program trace function.
TRON		√	√		√	√			Turns on the program trace function.

Sequencing Commands and Functions

Command	Level I	Level II	TRSDOS	TRSDOS	TAPE	TRSDOS	8K COLOR	POCKET	Description
END	√	√	√	√	√	√	√	√	ENDs program execution.
ERROR(N)		√	√	√	√	√			Simulates the specified error.
EXEC						√			Transfers control to machine language program.
FOR-NEXT	√	√	√	√	√	√	√	√	Program loop.
GOSUB	√	√	√	√	√	√	√	√	Transfers program control to specified subroutine.
GOTO	√	√	√	√	√	√	√	√	Transfers program control to the specified line.
NEXT	√	√	√	√	√	√	√	√	Ends FOR-NEXT loop.
ON	√	√	√	√	√	√			Multi-way branch used with GOTO and GOSUB.
ON ERROR GOTO			√	√	√	√			Sets up an error-handling routine.
ON ERROR GOTO 0		√	√	√	√	√			Disables an error handling-routine.
RESUME		√	√	√	√	√			Ends an error-handling routine.
RETURN	√	√	√	√	√	√	√	√	Returns from subroutine to statement after GOSUB
STEP	√	√	√	√	√	√	√	√	Increments or decrements FOR-NEXT loop index.
STOP	√	√	√	√	√	√	√	√	Stops execution of a program.
SYSTEM			√		√	√			Executes TRSDOS command, returns to BASIC.
TO	√	√	√	√	√	√	√	√	Used to specify index range in FOR-NEXT statement.

These BASIC instructions are typical. The BASIC you use may differ from this set. If so, mark the necessary changes directly on this reference sheet.

Instructions

AUTO *start, increment*   Numbers lines automatically.
AUTO   AUTO 150, 20   AUTO ,5

CLEAR*n*   Reserves n bytes of string storage space; initializes all variables.
CLEAR   CLEAR 75   CLEAR 0

CLOAD   Loads BASIC program file from cassette. Only the first character of the file name is used.
CLOAD   CLOAD "MIXIT"

CLOAD?   Compares program on tape byte-for-byte with resident program.
CLOAD?   CLOAD? "MIXIT"

CLS   Clears the display.
CLS

CONT   Continues execution of program after BREAK or STOP.
CONT

CSAVE   Stores resident program on cassette tape. A file name is required. Only the first character of the file name is used.
CSAVE "MIXIT"

DATA   Stores data to be accessed by a READ statement.
DATA "LINCOLN, A.", 1861, ILLINOIS

DEFDBL   Defines variables as double-precision.
DEFDBL V, X-Z

DEFINT   Defines variables as integer type.
DEFINT A, I-N

DEFSNG   Defines variables as single-precision.
DEFSNG I, W-Z

DEFSTR   Defines variables as string type.
DEFSTR C, L-Z

DELETE   Erases program lines from memory.
DELETE 1205   DELETE − 80   DELETE.

DIM   Dimensions one or more arrays.
DIM R(75), W(40)   DIM AR$(8 25)
DIM L%(3, 18, 5)

EDIT   Puts computer into edit mode for specified line.
See **Edit Commands.**
EDIT 100   EDIT.

END   Ends program execution.
END

ERROR*(n)*   Simulates the specified, error n = 1 − 23.
ERROR(1)

FOR ... TO ... STEP/NEXT   Opens program loop.
FOR I = 1 TO 8 (...) NEXT I
FOR C! = 0 TO 5 STEP .2 (...) NEXT C!

GOSUB   Transfers program control to the specified subroutine.
GOSUB 750

GOTO   Transfers program control to the specified line.
GOTO 180

IF ... THEN ... ELSE   Tests conditional expression.
IF P = Q THEN 200
IF N% < 0 THEN 150 ELSE N% = N% − 1

INPUT   Inputs data from keyboard.
INPUT X≠   INPUT L, M, N   INPUT "NEXT";N

INPUT ≠− *1*   Inputs data from cassette.
INPUT ≠ − 1, A

LET   Assigns value to variable (optional).
LET X = 7.05   LET R2 = R1   LET C$ = "RED"

LIST   Lists program lines to the video display.
LIST   LIST 50-85

LLIST   Lists program lines to the video display.
LLIST   LLIST 50−

LPRINT   Prints an item or list of items on the printer.
LPRINT CAP$; "IS THE CAPITAL OF"; ST$

LPRINT TAB   Moves printer carriage to specified position
LPRINT TAB(25) "NAME"

LPRINT USING   Prints formatted numbers and strings on the printer. See PRINT USING for list of field specifiers.
LPRINT USING "≠ ≠ ≠ ≠,"; 1234

NEW   Erases program from memory; initializes all variables.
NEW

ON ERROR GOTO   Sets up an error-handling routine.
ON ERROR GOTO 2100

ON ERROR GOTO 0   Disables an error-handling routine.
ON ERROR GOTO 0

ON . . . GOSUB   Multi-way branch to specified subroutines.
ON Y GOSUB 50, 100, 150, 200

ON . . . GOTO   Multi-way branch to specified lines.
ON X GOTO 190, 200, 210

OUT *p, v*   Sends value to specified port. p and v = 0 − 255.
OUT 255, 0

POKE *n, v,*   Puts value v (0 − 255) into location n (15360 to end of memory). **See POKE Addresses.**
POKE 15872, 255

PRINT   Prints an item or list of items on the display at current cursor position.
PRINT X! + Y!   PRINT "U.S.A."

PRINT @*n*   Prints beginning at n, n = 0 − 1023.
PRINT @ 477, "CENTER"

PRINT #− *1*   Writes data to cassette.
PRINT # − 1, A

PRINT TAB   Moves cursor right to specified tab position.
PRINT TAB(20) "NAME"

PRINT USING   Formats strings and numbers:
Formats numbers.
PRINT USING "# # # # #"; 66.2
.   Decimal point.
PRINT USING "# #.#"; 58.76
,   Displays comma to left of every third digit.
PRINT USING "# # # #,"; 1234
**   Fills leading spaces with asterisks.
PRINT USING "*# # # #"; 44.0
$$ Floating dollar sign.
PRINT USING "$$# #.# #"; 118.6735
**$ Floating dollar sign; fills leading spaces with asterisks.
PRINT USING "**$#.# #"; 8.333
(Exponential format. Press ⬆ ; to generate this character.
PRINT USING "# #.#(((("; 8527100
+   In first position, causes sign to be printed; in last position causes sign to be printed after the number.
PRINT USING "+# # #"; −216
−   Minus sign after negative numbers, space after positive.
PRINT USING "# # #.# −"; −8124.420
!   Returns first string character.
PRINT USING "!"; "YELLOW"

%spaces%   String field; length of field is number of spaces plus 2.
PRINT USING "% %"; "BLUE"

RANDOM   Reseeds random number generator.
RANDOM

READ   Reads value(s) from a DATA statement.
READ T   READ S$ READ NM$, AGE

REM   Remark; instructs computer to ignore rest of line. Is an abbreviation for :REM.
REM PLACE COMMENTS HERE   'HERE TOO

RESET *(x, y)*   Turns off graphics block at specified location. x (horizontal) = 0 − 127; y (vertical) = 0 − 47.
RESET (21, 40)   RESET (L1, L2)

RESTORE   Resets data pointer to first item in first data line.
RESTORE

RESUME   Ends an error-handling routine by specifying where normal execution is to resume.
RESUME   RESUME 40   RESUME NEXT

RETURN   Returns from subroutine to next statement after GOSUB.
RETURN

RUN   Executes resident program or portion of it.
RUN   RUN 150

SET *(x, y)*   Turns on graphics block at specified location. x (horizontal) = 0 − 127; y (vertical) = 0 − 47.
SET (10, 0)   SET (L1, L2)

STOP   Stops execution of a program.
STOP

SYSTEM   Puts computer in monitor mode, allows loading of object files. In response to *?, type filename or /address.
SYSTEM

TROFF   Turns off the trace.
TROFF

TRON   Turns on the trace.
TRON

224

Error Messages

Abbreviation	Explanation
NF	NEXT without FOR
SN	Syntax error
RG	RETURN without GOSUB
OD	Out of data
FC	Illegal function call
OV	Overflow
OM	Out of memory
UL	Undefined line
BS	Subscript out of range
DD	Redimensioned array
/0	Division by zero
ID	Illegal direct
TM	Type mismatch
OS	Out of string space
LS	String too long
ST	String formula too complex
CN	Can't continue
NR	No RESUME
RW	RESUME without error
UE	Undefined error
MO	Missing operand
FD	Bad file data
L3	Disk BASIC feature

Control Keys

Key	Description
◄	Cancels last character typed; moves cursor back one space.
SHIFT ◄	Erases current line.
BREAK	Interrupts anything in progress and returns to command level.
CLEAR	Clears the screen.
ENTER	Signifies end of current line.
SPACEBAR	Enters a space (blank) character and moves cursor one space forward.
►	Advances cursor to next tab position.
SHIFT ►	Puts display in 32-character mode.
↓	Line feed and carriage return.
SHIFT ↓	"Control" key—hold down these two and press any key A-Z for control A-control Z.
SHIFT ↓ ✱	Copies the display contents to the printer.
SHIFT @	Causes currently executing program to pause (press any key to continue).

Edit Commands

Command	Description
A	Cancels changes and starts again.
n C	Changes n characters
n D	Deletes n characters.
E	Ends editing and saves all changes.
H	Hacks line and inserts at end.
I	Inserts characters.
n K c	Kills all characters up to nth occurrence of c.
L	Lists the line.
Q	Quits edit mode and cancels all changes.
n S c	Searches for nth occurrence of c.
X	Extends line (inserts at end).
SHIFT ↑	Causes escape from command.
ENTER	Records all changes and exits edit mode.
n SPACEBAR	Moves cursor n spaces to the right.
n ◄	Moves cursor n spaces to the left.

Special Characters

Character	Description
'	Abbreviation for :REM
%	Makes variable integer-precision.
!	Makes variable single-precision.
#	Makes variable double-precision.
$	Makes variable string type.
:	Separates statements on the same line.
?	Same as PRINT (but L? can't be substituted for LPRINT).
'	PRINT punctuation: spaces over to the next 16-column PRINT zone.
;	PRINT punctuation: separates items in a PRINT list but does not add spaces when they are outpu

Functions

Argument ranges are indicated below by special letters:

x: $(-1 \times 10E\ 38, -1 \times 10E - 38)$,
      $(1 \times 10E - 38, 1 \times 10E\ 38)$

c: (0,255)

n: (−32768, 32767)

str: string argument

var: variable name

ABS(x)   Computes absolute value.
    Y = ABS(X)

ASC(str)   Returns ASCII code of first character of string.
    A = ASC(T$)

ATN(x)   Computes arctangent; value returned in radians.
    Y = ATN(X/3)

CDBL(x)   Converts to double-precision.
    X# = CDBL(N*3)

CHR$(c)   Returns character for ASCII, control, or graphics code.
    P$ = CHR$(T)

CINT(n)   Returns largest integer not greater than n.
    PRINT CINT (15.0075)

COS(x)   Computes cosine; angle must be in radians.
    Y = COS(X)

CSNG(x)   Converts to single-precision.
    FC = CSNG(TM#)

ERL   Returns the line number in which an error has occurred.
    PRINT ERL

ERR   If an error occurs, returns a value related to the error code: value returned = (error code − 1) ·2.
    IF ERR = 12 THEN 650 ELSE 800

EXP(x)   Computes natural antilog.
    Y = EXP(X)

FIX(x)   Truncates all digits to right of decimal point.
    Y = FIX(X)

FRE(numeric)   Find amount of free memory.
    F = FRE(X)   PRINT FRE(10)

FRE(str)   Returns amount of unused string space. str is any string constant or string variable.
    FRE("C")   FRE(C$)

INKEY$ Gets keyboard character if available.
    A$ = INKEY$

INP(p)   Gets value from specified port. p = 0− 255.
    V = INP(255)

INT(x)   Returns largest whole number not greater than x.
    Y = INT(X)

LEFT$(str, c)   Returns left portion of string.
    P$ = LEFT$(M$, 7)

LEN(str)   Returns the number of characters in a string.
    X = LEN(SEN$)

LOG(x)   Computes natural logarithm.
    Y = LOG(X)

MEM   Finds amount of free memory.
    PRINT MEM

MID$ (string, pos, len)   Returns a substring of another string. If length option is omitted, the entire string right of pos is returned.
    PRINT MID$(A$, 3, 2)   F$ = MID$ (A$, 3)

PEEK (n)   Gets value in location n (n = 0 to end of memory).
    V = PEEK (18520)

POINT(x, y)   Tests whether specified graphics block is on or off. x (horizontal) = 0 − 127; y (vertical) = 0 − 47.
    IF POINT (13, 35) THEN PRINT "ON" ELSE PRINT "OFF"

POS(x)   Returns column position of cursor (0 − 63). x is a dummy argument.
    PRINT TAB(40) POS(0)

RIGHT$(str, c)   Returns right portion of string.
    ZIP$ = RIGHT(AD$, 5)

RND(n)   Generates a "random" number between 1 and n if n > 1, or between 1 if n = 0.
    Y = RND(100)   PRINT RND(0)   R = RND(X)

SGN(x)   Returns sign component: − 1, 0, 1 if x is negative, zero, positive.
    X = SGN (A·B)

SIN(x)   Computes sine; angle must be in radians.
    Y = SIN(X)

SQR(x)   Computes square root.
    Y = SQR (A + B)

STR$(x)   Converts a numeric expression to a string.
    S$ = STR$(X)

STRING$(l, c)   Returns string of characters of length l. Character c can be specified as an ASCII code or as a string.
    B$ = (125, "?")   B$ = STRING$ (125, 63)

TAN(x)   Computes tangent; angle must be in radians.
    Y = TAN(X)

TIME$ Returns the time (in 24-hour format) and the data as a 17-character string.
    A$ = TIME$

USR(x)   Calls a machine-language subroutine whose address is stored at 16526 − 16527.
    PRINT USR(− 1)   X = USR(Y)

VAL(str)   Evaluates a string as a number.
    V% = VAL("100 DOLLARS")

VARPTR(var)   Gets address where variable contents are stored.
    Y = USR (VARPTR (X))

INDEX

W

Instructions

AUTO *start, increment*   Numbers lines automatically.
AUTO   AUTO 150, 20   AUTO ,5

CLEAR*n*   Reserves n bytes of string storage space;
initializes all variables.
CLEAR   CLEAR 75   CLEAR 0

CLOAD   Loads BASIC program file from cassette. Only
the first character of the file name is used.
CLOAD   CLOAD "MIXIT"

CLOAD?   Compares program on tape byte-for-byte
with resident program.
CLOAD?   CLOAD? "MIXIT"

CLS   Clears the display.
CLS

CONT   Continues execution of program after BREAK
or STOP.
CONT

CSAVE   Stores resident program on cassette tape. A file
name is required. Only the first character of the file name is
used.
CSAVE "MIXIT"

DATA   Stores data to be accessed by a READ state-
ment.
DATA "LINCOLN, A", 1861, ILLINOIS

DEFDBL   Defines variables as double-precision.
DEFDBL V, X-Z

DEFINT   Defines variables as integer type.
DEFINT A, I-N

DEFSNG   Defines variables as single-precision.
DEFSNG I, W-Z

DEFSTR   Defines variables as string type.
DEFSTR C, L-Z

DELETE   Erases program lines from memory.
DELETE 1205   DELETE – 80   DELETE.

DIM   Dimensions one or more arrays.
DIM R(75), W(40)   DIM ARS(8 25)
DIM L%(3, 18, 5)

EDIT   Puts computer into edit mode for specified line.
See Edit Commands.
EDIT 100   EDIT.

END   Ends program execution.
END

ERROR(*n***)**   Simulates the specified, error n   1   23
ERROR(1)

FOR . . . TO . . . STEP/NEXT   Opens program loop
FOR I   1 TO 8 (. . .) NEXT I
FOR C!   0 TO 5 STEP .2 (. . .) NEXT C!

GOSUB   Transfers program control to the specified
subroutine.
GOSUB 750

GOTO   Transfers program control to the specified line
GOTO 180

IF . . . THEN . . . ELSE   Tests conditional expression
IF P   Q THEN 200
IF N% < 0 THEN 150 ELSE N%   N%   1

INPUT   Inputs data from keyboard.
INPUT X#   INPUT L, M, N   INPUT "NEXT",N

INPUT *# 1*   Inputs data from cassette.
INPUT #   1, A

LET   Assigns value to variable (optional).
LET X   7.05   LET R2   R1   LET C$ "RED"

LIST   Lists program lines to the video display.
LIST   LIST 50-85

LLIST   Lists program lines to the video display.
LLIST   LLIST 50—

LPRINT   Prints an item or list of items on the printer
LPRINT CAPS; "IS THE CAPITAL OF"; STS

LPRINT TAB   Moves printer carriage to specified posi-
tion
LPRINT TAB(25) "NAME"

LPRINT USING   Prints formatted numbers and strings
on the printer. See PRINT USING for list of field specifiers
LPRINT USING "# # # #,"; 1234

NEW   Erases program from memory; initializes all
variables.
NEW

ON ERROR GOTO   Sets up an error-handling routine
ON ERROR GOTO 2100

ON ERROR GOTO 0   Disables an error-handling
routine.
ON ERROR GOTO 0

ON . . . GOSUB   Multi-way branch to specified subrou-
tines.
ON Y GOSUB 50, 100, 150, 200

ON . . . GOTO   Multi-way branch to specified lines.
ON X GOTO 190, 200, 210

OUT *p, v*   Sends value to specified port. p and v =
0   255.
OUT 255, 0

POKE *n, v,*   Puts value v (0 – 255) into location n (15360
to end of memory). **See POKE Addresses.**
POKE 15872, 255

PRINT   Prints an item or list of items on the display at
current cursor position.
PRINT X! – Y!   PRINT "U.S.A."

PRINT *(a n*   Prints beginning at n, n – 0 – 1023.
PRINT @ 477, "CENTER"

PRINT *# – 1*   Writes data to cassette.
PRINT #   1, A

PRINT TAB   Moves cursor right to specified tab posi-
tion.
PRINT TAB(20) "NAME"

PRINT USING   Formats strings and numbers:
\#   Formats numbers.
    PRINT USING "# # # #"; 66.2
.   Decimal point.
    PRINT USING "# #.#"; 58.76
,   Displays comma to left of every third digit.
    PRINT USING "# # # #,"; 1234
**   Fills leading spaces with asterisks.
    PRINT USING "**# # # #"; 44.0
\$\$ Floating dollar sign.
    PRINT USING "$$# # #"; 118.6735
**\$ Floating dollar sign; fills leading spaces with asterisks.
    PRINT USING "**$# #"; 8.333
(Exponential format. Press ↑ ; to generate this character.
    PRINT USING "# #.#((((" ; 8527100
+   In first position, causes sign to be printed; in last position
causes sign to be printed after the number.
    PRINT USING "+# # #"; –216
    Minus sign after negative numbers, space after positive.
    PRINT USING "# # # # #"; –8124.420
!   Returns first string character.
    PRINT USING "!"; "YELLOW"

%spaces%   String field; length of field is number of spaces
plus 2.
PRINT USING "% %"; "BLUE"

RANDOM   Reseeds random number generator.
RANDOM

READ   Reads value(s) from a DATA statement.
READ T   READ SS   READ NMS, AGE

REM   Remark; instructs computer to ignore rest of line. Is
an abbreviation for :REM.
REM PLACE COMMENTS HERE   'HERE TOO

RESET *(x, y)*   Turns off graphics block at specified
location. x (horizontal) – 0 – 127; y (vertical) – 0 – 47.
RESET (21, 40)   RESET (L1, L2)

RESTORE   Resets data pointer to first item in first data
line.
RESTORE

RESUME   Ends an error-handling routine by specifying
where normal execution is to resume.
RESUME   RESUME 40   RESUME NEXT

RETURN   Returns from subroutine to next statement
after GOSUB.
RETURN

RUN   Executes resident program or portion of it.
RUN   RUN 150

SET *(x, y)*   Turns on graphics block at specified loca-
tion. x (horizontal) – 0 – 127; y (vertical) – 0 – 47.
SET (10, 0)   SET (L1, L2)

STOP   Stops execution of a program.
STOP

SYSTEM   Puts computer in monitor mode, allows
loading of object files. In response to *?, type filename or
/address.
SYSTEM

TROFF   Turns off the trace.
TROFF

TRON   Turns on the trace.
TRON

Error Messages

Abbreviation	Explanation
NF	NEXT without FOR
SN	Syntax error
RG	RETURN without GOSUB
OD	Out of data
FC	Illegal function call
OV	Overflow
OM	Out of memory
UL	Undefined line
BS	Subscript out of range
DD	Redimensioned array
/0	Division by zero
ID	Illegal direct
TM	Type mismatch
OS	Out of string space
LS	String too long
ST	String formula too complex
CN	Can't continue
NR	No RESUME
RW	RESUME without error
UE	Undefined error
MO	Missing operand
FD	Bad file data
L3	Disk BASIC feature

Control Keys

Key	Function
←	Cancels last character typed; moves cursor back one space.
SHIFT ←	Erases current line.
BREAK	Interrupts anything in progress and returns to command level.
CLEAR	Clears the screen.
ENTER	Signifies end of current line.
SPACEBAR	Enters a space (blank) character and moves cursor one space forward.
→	Advances cursor to next tab position.
SHIFT →	Puts display in 32-character mode.
↑	Line feed and carriage return.
SHIFT ↑	"Control" key—hold down these two and press any key A-Z for control A-control Z.
SHIFT ↑ ✳	Copies the display contents to the printer.
SHIFT @	Causes currently executing program to pause (press any key to continue).

Edit Commands

Command	Function
A	Cancels changes and starts again.
n C	Changes n characters.
n D	Deletes n characters.
E	Ends editing and saves all changes.
H	Hacks line and inserts at end.
I	Inserts characters.
n K c	Kills all characters up to nth occurrence of c.
L	Lists the line.
Q	Quits edit mode and cancels all changes.
n S c	Searches for nth occurrence of c.
X	Extends line (inserts at end).
SHIFT ↓	Causes escape from command.
ENTER	Records all changes and exits edit mode.
n SPACEBAR	Moves cursor n spaces to the right.
n →	Moves cursor n spaces to the left.

Special Characters

Char	Function
'	Abbreviation for :REM
%	Makes variable integer-precision.
!	Makes variable single-precision.
#	Makes variable double-precision.
$	Makes variable string type.
:	Separates statements on the same line.
?	Same as PRINT (but L? can't be substituted for LPRINT).
,	PRINT punctuation: spaces over to the next 16-column PRINT zone.
;	PRINT punctuation: separates items in a PRINT list but does not add spaces when they are output.